Ward-Takahashi identity, soft photon theorem and the magnetic moment of the Δ resonance. [1]

A. I. Machavariani[a b c] **and Amand Faessler** [a]

[a] Institute für Theoretische Physik der Univesität Tübingen,
Tübingen D-72076, Germany

[b] Joint Institute for Nuclear Research, Dubna, Moscow region 141980, Russia

[c] High Energy Physics Institute of Tbilisi State University, University str. 9, Tbilisi 380086, Georgia

Abstract

Starting from the Ward-Takahashi identity for the radiative πN scattering amplitude a generalization of the soft photon theorem approach is obtained for an arbitrary energy of an emitted photon. The external particle radiation part of the $\pi N \to \gamma' \pi' N'$ amplitude is analytically reduced to the double Δ exchange amplitude with the $\Delta \to \gamma' \Delta'$ vertex function.

We have shown, that the double Δ exchange amplitudes with internal Δ radiation is connected by current conservation with the corresponding part of the external particle radiation terms. Moreover according to the current conservation the internal and external particle radiation terms with the $\Delta - \gamma' \Delta'$ vertex have a opposite sign i.e. they must cancel each other. Therefore we have a screening of the internal double Δ exchange diagram with the $\Delta - \gamma' \Delta'$ vertex by the external particle radiation. This enables us to obtain a model independent estimation of the dipole magnetic moment of Δ^+ and Δ^{++} resonances μ_Δ through the anomalous magnetic moment of the proton μ_p as $\mu_{\Delta^+} = \frac{M_\Delta}{m_p}\mu_p$ and $\mu_{\Delta^{++}} = \frac{3}{2}\mu_{\Delta^+}$ in agreement with the values obtained from the fit of the experimental cross section of the $\pi^+ p \to \gamma' \pi^+ p$ reaction.

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1. INTRODUCTION

The bremsstrahlung reactions in the low and intermediate energy region (up to 1GeV) are often investigated via the low energy photon theorem [1]-[10]. This approach enables us to calculate the bremsstrahlung amplitude in an expansion in powers of the small momentum k' of the emitted photon $A = a + b/k' + ck'^2 + ...$, where the first two terms can be reproduced exactly with the corresponding non-radiative amplitudes. A starting point for the low energy theorems for the bremsstrahlung reactions are the external particle radiation diagrams (Fig. 1) which determine the infrared behavior of this reaction. Using the current conservation condition one obtains a model independent connection between the external and internal particle radiation terms. Corresponding internal particle radiation terms allow to extract the values of the resonances from the experimental data. However the prescription for the construction of the bremsstrahlung amplitude in the low energy limit $k' \to 0$ is not unique and there ambiguities appear from the soft photon approach [7].

In this paper we study the pion-nucleon bremsstrahlung based on the the Ward-Takahashi identity, because this identity generates the current conservation for the on mass shell bremsstrahlung amplitude $k'_\mu A^\mu = 0$ using the equal-time commutators between the photon current operator and external particle field operators. On the other hand these equal-time commutator relations follow from charge conservation. In this way the current conservation for the full bremsstrahlung amplitude $k'_\mu A^\mu = 0$ reduces to the current conservation for the external particle radiation part $\mathcal{E}^\mu$, i.e. $k'_\mu A^\mu = k'_\mu \mathcal{E}^\mu + B = 0$, where B is a combination of the non-radiative off mass shell scattering amplitudes. This form of the current conservation condition is exactly the same as in the low energy theorems. But the present approach is not restricted to the low energy limit of the final photon and it can be applied for other reactions with electromagnetic interactions like pion photo-production, Compton scattering etc. Current conservation in this approach provides a model-independent connection between the external $\mathcal{E}^\mu$ and internal $\mathcal{I}^\mu$ particle radiation terms. Moreover, the mechanism of current conservation indicates, that the corresponding set of the external and the internal particle radiation terms have the opposite sign, i. e. they must cancel. Thus in the considered reactions a screening of the internal particle radiation terms by the external particle radiation diagrams must be observed.

The suggested approach is applied to the radiative πN scattering reaction in the Δ resonance region. Using the projections on the spin 3/2 amplitudes with the Δ exchange propagators, from the external particle radiation part of the πN bremsstrahlung amplitude the double Δ exchange part with the $\Delta - \gamma'\Delta'$ vertex is exactly extracted. This part $\widetilde{\mathcal{H}}^\mu$ is connected with the internal Δ radiation term H^μ through current conservation as $k'_\mu A^\mu = k'_\mu \widetilde{\mathcal{H}}^\mu + k'_\mu \mathrm{H}^\mu = 0$. This enables us to obtain a model independent relations between the $\Delta - \gamma'\Delta'$ vertex functions in H^μ and in $\widetilde{\mathcal{H}}^\mu$ and to determine the magnetic dipole moment μ_Δ of the Δ via the anomalous magnetic moment of the proton μ_p.

This paper consists of four sections and two appendixes. In the next section the current conservation conditions for the bremsstrahlung amplitude are derived using the Ward-

Takahashi identity. In Sect. 3 an analytical extraction of the double Δ exchange diagram from the external particle radiation terms is given. Using current conservation this term is combined with the internal Δ radiation diagram (Fig.2B). This model independent relation between the internal and external particle radiation terms enables us to determine the magnetic dipole momenta of Δ^+ and Δ^{++} through the anomalous magnetic moment of the proton. The conclusions are presented in Sect. 4. Appendix A and appendix B give the formulas for projection on the intermediate spin 3/2 states and for the $\Delta - \gamma'\Delta'$ vertex functions.

2. Ward-Takahashi identities for the pion-nucleon bremsstrahlung amplitude

We consider the radiative pion-nucleon scattering

$$\pi(p_\pi) \ + \ N(p_N) \Longrightarrow \gamma'(k') \ + \ \pi'(p'_\pi) \ + \ N'(p'_N)$$

with on mass shell momenta of the pi-meson ($p_\pi = (\sqrt{\mathbf{p}_\pi^2 + m_\pi^2}, \mathbf{p}_\pi)$, $p'_\pi = (\sqrt{\mathbf{p'}_\pi^2 + m_\pi^2}, \mathbf{p'}_\pi)$, nucleon ($p_N = (\sqrt{\mathbf{p}_N^2 + m_N^2}, \mathbf{p}_N)$, $p'_N = (\sqrt{\mathbf{p'}_N^2 + m_N^2}, \mathbf{p'}_N)$ and final photon ($k'^2 = 0$). In the physically interesting case the energy-momentum of the final photon is $k'_\mu = (p_N + p_\pi - p'_\pi - p'_N)_\mu$.

Following the derivation of the Ward-Takahashi identities (see e.g. ch. **8.4.1** in the Itzykson and Zuber book[13]) we start with the on shell amplitude $A^\mu_{\gamma'\pi'N'-\pi N}$

$$k'_\mu A^\mu_{\gamma'\pi'N'-\pi N}(\mathbf{p'_\pi}, \mathbf{p'_N}, \mathbf{k'}; \mathbf{p_\pi}, \mathbf{p_N}) = \overline{u}(\mathbf{p'}_N)(\gamma_\nu {p'_N}^\nu - m_N)({p'_\pi}^2 - m_\pi^2) k'_\mu \tau^\mu (\gamma_\nu {p_N}^\nu - m_N)({p_\pi}^2 - m_\pi^2) u(\mathbf{p}_N), \tag{2.1}$$

where the Green function τ^μ is expressed via the photon source operator $\mathcal{J}^\mu(z)$ and the pion and the nucleon field operators $\Phi(x)$ and $\Psi(y)$ as

$$k'_\mu \tau^\mu = i \int d^4z d^4y' d^4x' d^4y d^4x e^{ik'z + ip'_\pi x' + ip'_N y' - ip_\pi x - ip_N y} \frac{\partial}{\partial z^\mu} < 0|\mathsf{T}\Big(\Psi(y')\Phi(x')\mathcal{J}^\mu(z)\overline{\Psi}(y)\Phi^+(x)\Big)|0> . \tag{2.2a}$$

Next we will use the well known relation for the time-ordered product of the quantum field operators

$$\frac{\partial}{\partial z^\mu} < 0|\mathsf{T}\Big(\Psi(y')\Phi(x')\mathcal{J}^\mu(z)\overline{\Psi}(y)\Phi^+(x)\Big)|0> = < 0|\mathsf{T}\Big(\Psi(y')\Phi(x')\Big(\frac{\partial}{\partial z^\mu}\mathcal{J}^\mu(z)\Big)\overline{\Psi}(y)\Phi^+(x)\Big)|0>$$
$$+\delta(z_o - x'_o) < 0|\mathsf{T}\Big(\Psi(y')\Big[\mathcal{J}^o(z), \Phi(x')\Big]\overline{\Psi}(y)\Phi^+(x)\Big)|0>$$
$$+\delta(z_o - y'_o) < 0|\mathsf{T}\Big(\Phi(x')\Big\{\mathcal{J}^o(z), \Psi(y')\Big\}\overline{\Psi}(y)\Phi^+(x)\Big)|0>$$
.
$$+\delta(z_o - x_o) < 0|\mathsf{T}\Big(\Psi(y')\Phi(x')\Big[\mathcal{J}^o(z), \Phi^+(x)\Big]\overline{\Psi}(y)\Big)|0>$$
$$+\delta(z_o - y_o) < 0|\mathsf{T}\Big((\Psi(y')\Phi(x')\Big\{\mathcal{J}^o(z), \overline{\Psi}(y)\Big\}\Phi^+(x)\Big)|0>$$
.

and employ the equal-time commutation conditions

$$\Big\{\mathcal{J}^o(z), \Psi(y')\Big\}\delta(z_o - y'_o) = -e_{N'}\delta^{(4)}(z - y')\Psi(y'); \quad \Big\{\mathcal{J}^o(z), \overline{\Psi}(y)\Big\}\delta(z_o - y_o) = e_N\delta^{(4)}(z - y)\overline{\Psi}(y) \tag{2.3a}$$

$$\Big[\mathcal{J}^o(z),\Phi(x')\Big]\delta(z_o-x'_o)=-e_{\pi'}\delta^{(4)}(z-x')\Phi(x');\quad \Big[\mathcal{J}^o(z),\Phi^+(x)\Big]\delta(z_o-x_o)=e_\pi\delta^{(4)}(z-x)\Phi^+(x),\tag{2.3b}$$

where $e_{N'}$, $e_{\pi'}$, e_N and e_π stand for the charge of the of the nucleons and pions in the final and initial states. In particular $e_N=1,0$ for proton and neutron, and $e_\pi=\pm 1,0$ for pi-mesons.

As result of these relations, eq.(2.2a) for the Green function takes a form

$$k'_\mu\tau^\mu=-i\int d^4y'd^4x'd^4yd^4xe^{ip'_\pi x'+ip'_N y'-ip_\pi+ip_N y}\Big(e_{N'}e^{ik'y'}+e_{\pi'}e^{ik'x'}-e_Ne^{ik'y}-e_\pi e^{ik'x}\Big)$$

$$<0|T\Big(\Psi(y')\Phi(x')\overline{\Psi}(y)\Phi^+(x)\Big)|0>.\tag{2.2b}$$

Equal-time commutators (2.3a,b) follow from the commutation relations between the electric charge operator $Q=\int d^3x\mathcal{J}^o(x)$ and the particle field operators with the charge e. These conditions express electric charge conservation for the local fields i.e. they represent one of the first principles in the quantum field theory.

Substituting expression (2.2b) into (2.1) we get

$$\begin{aligned}
k'_\mu A^\mu_{\gamma'\pi'N'-\pi N}(\mathbf{p}'_\pi,\mathbf{p}'_\mathbf{N},\mathbf{k}';\mathbf{p}_\pi,\mathbf{p}_\mathbf{N}) &= -i(2\pi)^4\ \delta^{(4)}(p'_N+p'_\pi+k'-p_\pi-p_N)\\
&\Big[\overline{u}(\mathbf{p}'_N)(\gamma_\nu {p'_N}^\nu-m_N)\frac{e_{N'}}{\gamma_\nu(p'_N+k')^\nu-m_N}<out;\mathbf{p}'_\pi|J(0)|\mathbf{p}_\pi\mathbf{p}_N;in>\\
&+({p'}^2_\pi-m^2_\pi)\frac{e_{\pi'}}{(p'_\pi+k')^2-m^2_\pi}<out;\mathbf{p}'_N|j_{\pi'}(0)|\mathbf{p}_\pi\mathbf{p}_N;in>\\
&-<out;\mathbf{p}'_\pi\mathbf{p}'_N|\overline{J}(0)|\mathbf{p}_\pi;in>\frac{e_N}{\gamma_\nu(p_N-k')^\nu-m_N}(\gamma_\nu {p_N}^\nu-m_N)u(\mathbf{p}_N)\\
&-<out;\mathbf{p}'_\pi\mathbf{p}'_N|j_\pi(0)|\mathbf{p}_N;in>\frac{e_\pi}{(p_\pi-k')^2-m^2_\pi}(p^2_\pi-m^2_\pi)\Big]
\end{aligned}\tag{2.4}$$

where $J(x)=(i\gamma_\nu\partial/\partial x_\nu-m_N)\Psi(x)$ and $j_\pi(x)=(\partial^2/\partial x^\nu\partial x_\nu-m^2_\pi)\Phi(x)$ denote the source operator of nucleon and pion.

For the on-mass shell external particles eq.(2.4) vanishes. In particular, for $k'=0$ this expression disappears due to cancellation of the on shell πN amplitudes in (2.4). Thus expression (2.4) presents the current conservation condition for the on-mass shell bremsstrahlung amplitude

$$k'_\mu\Big[A^\mu_{\gamma'\pi'N'-\pi N}(\mathbf{p}'_\pi,\mathbf{p}'_\mathbf{N},\mathbf{k}';\mathbf{p}_\pi,\mathbf{p}_\mathbf{N})\Big]_{on\ mass\ shell\ \pi',\ N',\ \pi,\ N}=0.\tag{2.5a}$$

In the off mass shell region for the four-momenta of the external pions and nucleons, where ${q'_\pi}^2 \neq m_\pi^2$ or ${q'_N}^2 \neq m_N^2$ or $q_\pi^2 \neq m_\pi^2$ or $q_N^2 \neq m_N^2$, eq.(2.4) is also valid, but the current conservation condition is violated

$$\left[{k'}_\mu A^\mu_{\gamma'\pi'N'-\pi N}(q'_\pi, q'_N; q_\pi, q_N) \right]_{off\ mass\ shell\ \pi',\ N',\ \pi,\ N} \neq 0. \tag{2.5b}$$

It is convenient to extract the full energy-momentum conservation δ function from the radiative πN scattering amplitude $A^\mu_{\gamma'\pi'N'-\pi N}$ and introduce the corresponding nonsingular amplitude $< out; \mathbf{p'}_N \mathbf{p'}_\pi | J^\mu(0) | \mathbf{p}_\pi \mathbf{p}_N; in >$

$${k'}_\mu A^\mu_{\gamma'\pi'N'-\pi N} = -i(2\pi)^4\ \delta^{(4)}(p'_N + p'_\pi + k' - p_\pi - p_N) {k'}_\mu < out; \mathbf{p'}_N \mathbf{p'}_\pi | J^\mu(0) | \mathbf{p}_\pi \mathbf{p}_N; in >, \tag{2.6}$$

Afterwards using the identity $a/(a+b) \equiv 1 - b/(a+b)$ in eq. (2.4) we obtain

$${k'}_\mu < out; \mathbf{p'}_N \mathbf{p}_{\pi'} | J^\mu(0) | \mathbf{p}_\pi \mathbf{p}_N; in >= \left[\mathcal{B}_{\gamma'\pi'N'-\pi N} + {k'}_\mu \mathcal{E}^\mu_{\gamma'\pi'N'-\pi N} \right]_{on\ mass\ shell\ \pi',\ N',\ \pi,\ N} = 0, \tag{2.7a}$$

$${k'}_\mu < {q'}_N {q'}_\pi | J^\mu(0) | q_\pi q_N >= \left[\mathcal{B}_{\gamma'\pi'N'-\pi N} + {k'}_\mu \mathcal{E}^\mu_{\gamma'\pi'N'-\pi N} \right|_{off\ mass\ shell\ \pi',\ or\ N',\ or\ \pi, or\ N} \neq 0, \tag{2.7b}$$

where

$$\mathcal{B}_{\gamma'\pi'N'-\pi N} = e_{N'} \overline{u}(\mathbf{p'}_N) < out; \mathbf{p'}_\pi | J(0) | \mathbf{p}_\pi \mathbf{p}_N; in > + e_{\pi'} < out; \mathbf{p'}_N | j_{\pi'}(0) | \mathbf{p}_\pi \mathbf{p}_N; in >$$
$$-e_N < out; \mathbf{p'}_\pi \mathbf{p'}_N | \overline{J}(0) | \mathbf{p}_\pi; in > u(\mathbf{p}_N) - e_\pi < out; \mathbf{p'}_\pi \mathbf{p'}_N | j_\pi(0) | \mathbf{p}_N; in >, \tag{2.8a}$$

$$\mathcal{E}^\mu_{\gamma'\pi'N'-\pi N} = -\Bigg[\overline{u}(\mathbf{p'}_N) \gamma^\mu \frac{\gamma_\nu (p'_N + k')^\nu + m_N}{2p'_N k'} e_{N'} < out; \mathbf{p'}_\pi | J(0) | \mathbf{p}_\pi \mathbf{p}_N; in >$$
$$+(2{p'}_\pi + k')^\mu \frac{e_{\pi'}}{2p'_\pi k'} < out; \mathbf{p'}_N | j_{\pi'}(0) | \mathbf{p}_\pi \mathbf{p}_N; in >$$
$$-e_N < out; \mathbf{p'}_\pi \mathbf{p'}_N | \overline{J}(0) | \mathbf{p}_\pi; in > \frac{\gamma_\nu (p_N - k')^\nu + m_N}{2p_N k'} \gamma^\mu u(\mathbf{p}_N)$$
$$- < out; \mathbf{p'}_\pi \mathbf{p'}_N | j_\pi(0) | \mathbf{p}_N; in > \frac{e_\pi}{2p_\pi k'} (2p_\pi - k')^\mu \Bigg] \tag{2.8b}$$

Amplitude $\mathcal{E}^\mu_{\gamma'\pi'N'-\pi N}$ describes the $\pi N \to \gamma'\pi'N'$ reaction with photon emission through the $N - \gamma'N'$ and $\pi - \gamma'\pi$ vertex functions in the tree approximation. $\mathcal{E}^\mu_{\gamma'\pi'N'-\pi N}$

(2.8b) can be described as a combination of the Feynman diagrams with external particle radiation as depicted on Fig.1. This external particle radiation amplitude is the starting formula for the derivation of the soft photon theorem [1]. Various applications of this method are given in [2, 3, 6, 7, 8]. Diagrams of Fig. 1 are responsible for the infrared behavior of the bremsstrahlung amplitude. The soft photon theorem enables us to restrict calculations of bremsstrahlung reactions to the leading diagrams on Fig. 1 and Fig. 2B which insures the validity of current conservation.

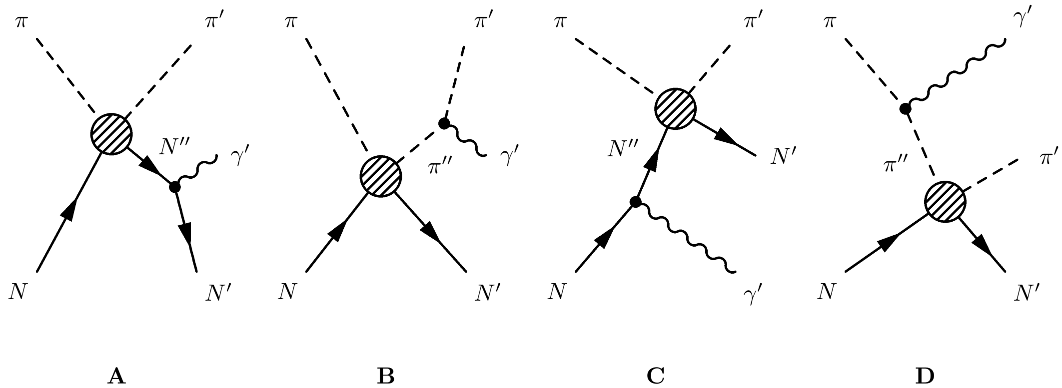
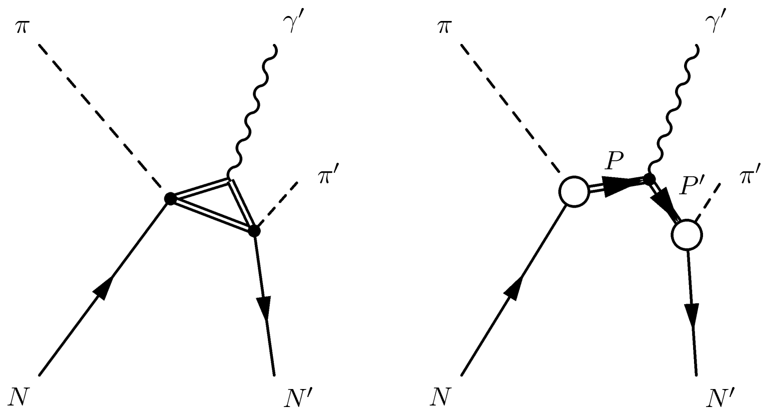


Figure 1: *Diagrams describing πN bremsstrahlung with photon emission by the external nucleons (A,C) and by the external pions (B,D) in $\mathcal{E}^{\mu}_{\gamma'\pi'N'-\pi N}$ (2.8b). The dashed circle indicates the off shell πN elastic scattering amplitudes (2.9a,b,c,d).*

The present derivation of current conservation (2.7a,b) is based on the Ward-Takahashi identity (2.2b) and is not restricted by the energy $k' = |\mathbf{k'}_{\gamma}|$ of the final photon. An interesting property is that the expressions (2.4) and (2.8b) contain the electromagnetic form factors of the external particles in the tree approximation. This is a result of the equal time commutation rules (2.3a,b) which follow from charge conservation. Thus current conservation (2.7a) is valid only for the on mass shell external particles and this condition is fulfilled for an arbitrary energy of the initial and final particles. But in the off mass shell region we get instead of (2.7a) the similar condition (2.7b) .

The πN scattering amplitudes in (2.8a,b) are functions of the three on-mass shell momenta from which one can construct only three independent Lorentz-invariant (Mandelstam) variables. Therefore we have

$$\overline{u}(\mathbf{p'}_N) < out; \mathbf{p'}_{\pi}|J(0)|\mathbf{p}_{\pi}\mathbf{p}_N; in >= \mathcal{T}_{N'}\Big((p'_{\pi}-p_{\pi}-p_N)^2; s, t_{\pi}\Big) = \mathcal{T}_{N'}\Big(m_N^2+2k'p'_N; s, t_{\pi}\Big) \tag{2.9a}$$



A **B**

Figure 2: *Diagram A presents a symbolic description of the internal particle radiation amplitude $\mathcal{I}^{\mu}_{\gamma'\pi'N'-\pi N}$ in eq.(3.17a). Diagram B describes the photon emission from the intermediate πN cluster with the four momentums P and P'.*

$$< out; \mathbf{p'}_N | j_{\pi'}(0) | \mathbf{p}_\pi \mathbf{p}_N; in >= \mathcal{T}_{\pi'}\Big((p'_N - p_\pi - p_N)^2; s, t_N \Big) = \mathcal{T}_{\pi'}\Big(m_\pi^2 + 2k'p'_\pi; s, t_N \Big) \quad (2.9b)$$

$$< out; \mathbf{p'}_\pi \mathbf{p'}_N | \overline{J}(0) | \mathbf{p}_\pi; in > u(\mathbf{p}_N) = \mathcal{T}_N\Big(s', t_\pi; (p'_\pi + p'_N - p_\pi)^2 \Big) = \mathcal{T}_N\Big(s', t_\pi; m_N^2 - 2k'p_N \Big) \quad (2.9c)$$

$$< out; \mathbf{p'}_\pi \mathbf{p'}_N | j_\pi(0) | \mathbf{p}_N; in >= \mathcal{T}_\pi\Big(s', t_N; (p'_\pi + p'_N - p_\pi)^2 \Big) = \mathcal{T}_\pi\Big(s', t_N; m_\pi^2 - 2k'p_\pi \Big). \quad (2.9d)$$

Amplitudes (2.9a,b,c,d) can be represented as a sum of the Feynman diagrams for the elastic πN scattering reactions, where three external particles being on mass shell and the four momentum of the fourth particle is determined via the energy-momentum conservation for the bremsstrahlung amplitude (2.6). The related invariant variable are

$$s' = (p'_N + p'_\pi)^2; \quad s = (p_N + p_\pi)^2 = (p'_N + p'_\pi + k')^2 = s' + 2k'(p'_N + p'_\pi) = s' + 2k'(p_N + p_\pi) \quad (2.10a)$$

$$t_N = (p'_N - p_N)^2; \quad t_\pi = (p'_\pi - p_\pi)^2 \quad (2.10b)$$

with the following relations between them

$$t_\pi + (p'_\pi - p_N)^2 + s = m_\pi^2 + 2m_N^2 + (p'_\pi - p_\pi - p_N)^2, \tag{2.11a}$$

$$t_N + (p'_N - p_\pi)^2 + s = m_N^2 + 2m_\pi^2 + (p'_N - p_\pi - p_N)^2, \tag{2.11b}$$

$$t_\pi + (p_\pi - p'_N)^2 + s' = m_\pi^2 + 2m_N^2 + (p_\pi - p'_\pi - p'_N)^2, \tag{2.11c}$$

$$t_N + (p_N - p'_\pi)^2 + s' = m_N^2 + 2m_\pi^2 + (p_N - p'_\pi - p'_N)^2. \tag{2.11d}$$

Next we represent amplitude (2.8b) in the time-ordered three-dimensional form which contains one on mass shell nucleon (Fig.1A,C) and one on mass shell pion (Fig.1B,D) exchange diagrams

$$\widetilde{\mathcal{E}}^\mu_{\gamma'\pi'N'-\pi N} = -\Bigg[\frac{\overline{u}(\mathbf{p}'_N)\Big[(2p'_N + k')^\mu - i\sigma^{\mu\nu}k'_\nu\Big]}{2p'_N k'}\frac{(p'_N + k')_\sigma\gamma^\sigma + m_N}{2m_N} e_{N'} < out; \mathbf{p}'_\pi|J(0)|\mathbf{p}_\pi\mathbf{p}_N; in >$$

$$+(2p'_\pi + k')^\mu \frac{e_{\pi'}}{2p'_\pi k'} < out; \mathbf{p}'_N|j_{\pi'}(0)|\mathbf{p}_\pi\mathbf{p}_N; in >$$

$$-e_N < out; \mathbf{p}'_\pi\mathbf{p}'_N|\overline{J}(0)|\mathbf{p}_\pi; in > \frac{(p_N - k')_\sigma\gamma^\sigma + m_N}{2m_N}\frac{\Big[(2p_N - k')^\mu - i\sigma^{\mu\nu}k'_\nu\Big]u(\mathbf{p}_N)}{2p'_N k'}$$

$$- < out; \mathbf{p}'_\pi\mathbf{p}'_N|j_\pi(0)|\mathbf{p}_N; in > \frac{e_\pi}{2p_\pi k'}(2p_\pi - k')^\mu\Bigg], \tag{2.12}$$

where relations $\overline{u}(\mathbf{p}'_N)\gamma^\mu(\gamma_\nu p'^{\ \nu}_N + m_N) = \overline{u}(\mathbf{p}'_N)2p'^{\ \mu}_N$ and $\overline{u}(\mathbf{p}'_N)\gamma^\mu\gamma^\nu k'_\nu = k'^\mu\overline{u}(\mathbf{p}'_N) - i\overline{u}(\mathbf{p}'_N)\sigma^{\mu\nu}k'_\nu\Big[u(\mathbf{p'_N} + \mathbf{k}')\overline{u}(\mathbf{p'_N} + \mathbf{k}') - v(\mathbf{p'_N} + \mathbf{k}')\overline{v}(\mathbf{p'_N} + \mathbf{k}')\Big]$ are used. For the sake of simplicity contributions of the antinucleon intermediate states in (2.12) are omitted. Therefore $\widetilde{\mathcal{E}}^\mu_{\gamma'\pi'N'-\pi N} \neq \mathcal{E}^\mu_{\gamma'\pi'N'-\pi N}$ and instead of (2.7a) we get

$$\Big[k'_\mu < out; \mathbf{p}'_N\mathbf{p}_{\pi'}|J^\mu(0)|\mathbf{p}_\pi\mathbf{p}_N; in >= \widetilde{\mathcal{B}}_{\gamma'\pi'N'-\pi N} + k'_\mu\widetilde{\mathcal{E}}^\mu_{\gamma'\pi'N'-\pi N}\Big]^{without\ \pi-\pi N\overline{N}\ transition}_{on\ mass\ shell\ \pi',\ N',\ \pi,\ N} = 0, \tag{2.13}$$

where

$$\widetilde{\mathcal{B}}_{\gamma'\pi'N'-\pi N} = e_{N'}\overline{u}(\mathbf{p}'_N)\frac{(p'_N + k')_\sigma\gamma^\sigma + m_N}{2m_N} < out; \mathbf{p}'_\pi|J(0)|\mathbf{p}_\pi\mathbf{p}_N; in > + e_{\pi'} < out; \mathbf{p}'_N|j_{\pi'}(0)|\mathbf{p}_\pi\mathbf{p}_N; in >$$

$$-e_N < out; \mathbf{p}'_\pi\mathbf{p}'_N|\overline{J}(0)|\mathbf{p}_\pi; in > \frac{(p_N - k')_\sigma\gamma^\sigma + m_N}{2m_N}u(\mathbf{p}_N) - e_\pi < out; \mathbf{p}'_\pi\mathbf{p}'_N|j_\pi(0)|\mathbf{p}_N; in >, \tag{2.14}$$

Conditions (2.7a), (2.8a,b) and (2.12) establish a relationships between the πN bremsstrahlung amplitude and the off mass shell elastic πN scattering amplitudes. The gauge terms in (2.12) are proportional to k'^{μ} and $\sigma^{\mu\nu}k'_{\nu}$. These terms modify the Green function τ^{μ} in (2.1). But for the on shell amplitude they can be ignored.

The gauge terms in $\mathcal{E}^{\mu}_{\gamma'\pi'N'-\pi N}$ (2.12) (Fig. 1) must contain contribution from the anomalous magnetic moment of the nucleon. For this aim one can use a gauge transformation $\gamma_{\mu} \Longrightarrow \gamma_{\mu} - i\mu/2m_N\sigma_{\mu\nu}k'^{\nu}$ which replaces $i\overline{u}(\mathbf{p'}_N)\sigma^{\mu\nu}k'_{\nu}u(\mathbf{p'}_N)$ via $i\mu_{N'}\overline{u}(\mathbf{p'}_N)\sigma^{\mu\nu}k'_{\nu}u(\mathbf{p'}_N)$. Then we obtain

$$\widetilde{\mathcal{E}}^{\mu}_{\gamma'\pi'N'-\pi N} = -\Bigg[\frac{\overline{u}(\mathbf{p'}_N)\Big[(2p'_N+k')^{\mu} - i\mu_{N'}\sigma^{\mu\nu}k'_{\nu}\Big]}{2p'_Nk'}\frac{(p'_N+k')_{\sigma}\gamma^{\sigma}+m_N}{2m_N}e_{N'} < out;\mathbf{p'}_{\pi}|J(0)|\mathbf{p}_{\pi}\mathbf{p}_N;in >$$

$$+(2p'_{\pi}+k')^{\mu}\frac{e_{\pi'}}{2p'_{\pi}k'} < out;\mathbf{p'}_N|j_{\pi'}(0)|\mathbf{p}_{\pi}\mathbf{p}_N;in >$$

$$-e_N < out;\mathbf{p'}_{\pi}\mathbf{p'}_N|\overline{J}(0)|\mathbf{p}_{\pi};in > \frac{(p_N-k')_{\sigma}\gamma^{\sigma}+m_N}{2m_N}\frac{\Big[(2p_N-k')^{\mu} - i\mu_N\sigma^{\mu\nu}k'_{\nu}\Big]u(\mathbf{p}_N)}{2p'_Nk'}$$

$$- < out;\mathbf{p'}_{\pi}\mathbf{p'}_N|j_{\pi}(0)|\mathbf{p}_N;in > \frac{e_{\pi}}{2p_{\pi}k'}(2p_{\pi}-k')^{\mu}\Bigg], \tag{2.15}$$

$\widetilde{\mathcal{E}}^{\mu}_{\gamma'\pi'N'-\pi N}$ (2.15) describes the radiative πN scattering amplitude with the external particle radiation, where the magnetic momenta of external nucleons determine the form of the gauge terms at $\sigma^{\mu\nu}k'_{\nu}$.

3. Internal and external particle radiation parts

It is convenient to introduce the total and relative momenta

$$P = p_{\pi}+p_N; \quad p = \frac{\alpha_{\pi}p_N - \alpha_N p_{\pi}}{\alpha_{\pi}+\alpha_N}; \quad p_N = \frac{\alpha_N P}{\alpha_{\pi}+\alpha_N} + p, \quad p_{\pi} = \frac{\alpha_{\pi}P}{\alpha_{\pi}+\alpha_N} - p, \tag{3.1a}$$

$$P' = p'_{\pi}+p'_N; \quad p' = \frac{\alpha'_{\pi}p'_N - \alpha'_N p'_{\pi}}{\alpha'_{\pi}+\alpha'_N}; \quad p'_N = \frac{\alpha'_N P'}{\alpha'_{\pi}+\alpha'_N} + p', \quad p'_{\pi} = \frac{\alpha'_{\pi}P'}{\alpha'_{\pi}+\alpha'_N} - p', \tag{3.1b}$$

where

$$\alpha_N = k'_{\nu}p^{\nu}_N, \quad \alpha_{\pi} = k'_{\nu}p^{\nu}_{\pi}; \quad \alpha'_N = k'_{\nu}{p'_N}^{\nu}, \quad \alpha'_{\pi} = k'_{\nu}{p'_{\pi}}^{\nu}. \tag{3.1c}$$

The relative moments p and p' are transverse to k'

$$k'_{\nu}p^{\nu} = 0; \quad k'_{\nu}p'^{\nu} = 0. \tag{3.2}$$

Therefore we can extract from $\widetilde{\mathcal{E}}^{\mu}_{\gamma'\pi'N'-\pi N}$ (2.15) the transverse part $\mathcal{C}^{\mu}_{\gamma'\pi'N'-\pi N}$

$$\widetilde{\mathcal{E}}^{\mu}_{\gamma'\pi'N'-\pi N} = \mathcal{M}^{\mu}_{\gamma'\pi'N'-\pi N} + \mathcal{C}^{\mu}_{\gamma'\pi'N'-\pi N} \tag{3.3a}$$

and

$$k'_{\mu}\mathcal{M}^{\mu}_{\gamma'\pi'N'-\pi N} = -\widetilde{\mathcal{B}}_{\gamma'\pi'N'-\pi N}; \quad k'_{\mu}\widetilde{\mathcal{E}}^{\mu}_{\gamma'\pi'N'-\pi N} = -\widetilde{\mathcal{B}}_{\gamma'\pi'N'-\pi N} \quad and \quad k'_{\mu}\mathcal{C}^{\mu}_{\gamma'\pi'N'-\pi N} = 0. \tag{3.3b}$$

Using (2.15) and (3.1a,b,c) we get

$$\mathcal{M}^{\mu}_{\gamma'\pi'N'-\pi N} = -\frac{1}{s-s'}\Big[\overline{u}(\mathbf{p'_N})\Big[2e_{N'}P'^{\mu} - i\mu_{N'}\sigma^{\mu\nu}k'_{\nu}\Big]u(\mathbf{p'_N}+\mathbf{k'})\overline{u}(\mathbf{p'_N}+\mathbf{k'}) < out; \mathbf{p'}_{\pi}|J(0)|\mathbf{p}_{\pi}\mathbf{p}_N; in >$$
$$- < out; \mathbf{p'}_{\pi}\mathbf{p'}_N|\overline{J}(0)|\mathbf{p}_{\pi}; in > u(\mathbf{p_N}-\mathbf{k'})\overline{u}(\mathbf{p_N}-\mathbf{k'})\Big[2e_N(P)^{\mu} - i\mu_N\sigma^{\mu\nu}k'_{\nu}\Big]u(\mathbf{p}_N)\Big]$$
$$-\frac{1}{s-s'}\Big[2e_{\pi'}P'^{\mu} < out; \mathbf{p'}_N|j_{\pi'}(0)|\mathbf{p}_{\pi}\mathbf{p}_N; in > -2e_{\pi}P^{\mu} < out; \mathbf{p'}_{\pi}\mathbf{p'}_N|j_{\pi}(0)|\mathbf{p}_N; in >\Big] \tag{3.4a}$$

$$\mathcal{C}^{\mu}_{\gamma'\pi'N'-\pi N} = -\Big[e_{N'}\frac{p'^{\mu}}{\alpha'_N}\overline{u}(\mathbf{p'}_N)u(\mathbf{p'_N}+\mathbf{k'})\overline{u}(\mathbf{p'_N}+\mathbf{k'}) < out; \mathbf{p'}_{\pi}|J(0)|\mathbf{p}_{\pi}\mathbf{p}_N; in >$$
$$-e_{\pi'}\frac{p'^{\mu}}{\alpha'_{\pi}} < out; \mathbf{p'}_N|j_{\pi'}(0)|\mathbf{p}_{\pi}\mathbf{p}_N; in > +e_{\pi}\frac{p^{\mu}}{\alpha_{\pi}} < out; \mathbf{p'}_{\pi}\mathbf{p'}_N|j_{\pi}(0)|\mathbf{p}_N; in >$$
$$-e_N\frac{p^{\mu}}{\alpha_N} < out; \mathbf{p'}_{\pi}\mathbf{p'}_N|\overline{J}(0)|\mathbf{p}_{\pi}; in > u(\mathbf{p_N}-\mathbf{k'})\overline{u}(\mathbf{p_N}-\mathbf{k'})u(\mathbf{p}_N)\Big] \tag{3.4b}$$

One can separate an other gauge term in (3.4b) introducing a new total four-momenta $P_{\pm}$ instead of P and P'

$$P_{\pm} = \frac{1}{2}(P \pm P'), \quad where\ P = P_+ + P_-; \quad P' = P_+ - P_- \quad and \quad P_-{}^{\mu} = \frac{1}{2}k'^{\mu}. \tag{3.5a}$$

This enables us to pick out another gauge term $\mathcal{F}^{\mu}_{\gamma'\pi'N'-\pi N}$

$$\mathcal{M}^{\mu}_{\gamma'\pi'N'-\pi N} = \mathcal{D}^{\mu}_{\gamma'\pi'N'-\pi N} + \mathcal{F}^{\mu}_{\gamma'\pi'N'-\pi N}, \tag{3.6}$$

where

$$\mathcal{D}^{\mu}_{\gamma'\pi'N'-\pi N} = -\frac{1}{s-s'}\Big[\overline{u}(\mathbf{p'_N})\Big[e_{N'}(P+P')^{\mu} - i\mu_{N'}\sigma^{\mu\nu}k'_{\nu}\Big]u(\mathbf{p'_N}+\mathbf{k'})\overline{u}(\mathbf{p'_N}+\mathbf{k'}) < out; \mathbf{p'}_{\pi}|J(0)|\mathbf{p}_{\pi}\mathbf{p}_N; in >$$

$$- < out; \mathbf{p'}_\pi \mathbf{p'}_N | \overline{J}(0) | \mathbf{p}_\pi; in > u(\mathbf{p_N} - \mathbf{k'}) \overline{u}(\mathbf{p_N} - \mathbf{k'}) \Big[e_N (P' + P)^\mu - i\mu_N \sigma^{\mu\nu} k'_\nu \Big] u(\mathbf{p}_N) \Big]$$

$$- \frac{1}{s - s'} \Big[e_{\pi'} (P + P')^\mu < out; \mathbf{p'}_N | j_{\pi'}(0) | \mathbf{p}_\pi \mathbf{p}_N; in > - e_\pi (P + P')^\mu < out; \mathbf{p'}_\pi \mathbf{p'}_N | j_\pi(0) | \mathbf{p}_N; in > \Big], \tag{3.7a}$$

$$\mathcal{F}^\mu_{\gamma'\pi'N'-\pi N} = \frac{k'^\mu}{s - s'} \Big[\overline{u}(\mathbf{p'}_N) \frac{(p'_N + k')_\sigma \gamma^\sigma + m_N}{2m_N} e_{N'} < out; \mathbf{p'}_\pi | J(0) | \mathbf{p}_\pi \mathbf{p}_N; in >$$

$$+ e_{\pi'} < out; \mathbf{p'}_N | j_{\pi'}(0) | \mathbf{p}_\pi \mathbf{p}_N; in >$$

$$+ e_N < out; \mathbf{p'}_\pi \mathbf{p'}_N | \overline{J}(0) | \mathbf{p}_\pi; in > \frac{(p_N - k')_\sigma \gamma^\sigma + m_N}{2m_N} u(\mathbf{p}_N) + e_\pi < out; \mathbf{p'}_\pi \mathbf{p'}_N | j_\pi(0) | \mathbf{p}_N; in > \Big], \tag{3.7b}$$

A projection on the spin 3/2 intermediate states in the $\gamma N - N$ vertex function, as shown in appendix A, generates the following redefinition of the expression (3.7a) $\mathcal{D}^\mu_{\gamma'\pi'N'-\pi N} \Longrightarrow \mathcal{H}^\mu_{\gamma'\pi'N'-\pi N}$

$$\mathcal{H}^\mu_{\gamma'\pi'N'-\pi N} = \frac{(p'_N)_a (p_N)_d (P + P')^\mu}{(p'_N . p_N)(s - s')} \overline{u}(\mathbf{p'_N}) i\gamma_5 u^a(\mathbf{P'_\Delta}) \Big\{ \overline{u}^b(\mathbf{P'_\Delta}) g_{bc} u^c(\mathbf{P_\Delta}) \Big\} \overline{u}^d(\mathbf{P_\Delta}) i\gamma_5 u(\mathbf{p_N})$$

$$\Big[\overline{u}(\mathbf{p_N}) \frac{(p'_N + k')_\sigma \gamma^\sigma + m_N}{2m_N} e_{N'} < out; \mathbf{p'}_\pi | J(0) | \mathbf{p}_\pi \mathbf{p}_N; in >$$

$$- e_N < out; \mathbf{p'}_\pi \mathbf{p'}_N | \overline{J}(0) | \mathbf{p}_\pi; in > \frac{(p_N - k')_\sigma \gamma^\sigma + m_N}{2m_N} u(\mathbf{p'_N})$$

$$+ \overline{u}(\mathbf{p_N}) u(\mathbf{p'_N}) e_{\pi'} < out; \mathbf{p'}_N | j_{\pi'}(0) | \mathbf{p}_\pi \mathbf{p}_N; in > - e_\pi < out; \mathbf{p'}_\pi \mathbf{p'}_N | j_\pi(0) | \mathbf{p}_N; in > \overline{u}(\mathbf{p_N}) u(\mathbf{p'_N}) \Big]$$

$$+ \frac{(p'_N)_a (p_N)_d}{(p'_N . p_N)(s - s')} \overline{u}(\mathbf{p'_N}) i\gamma_5 u^a(\mathbf{P'_\Delta}) \Big\{ \overline{u}^b(\mathbf{P'_\Delta}) g_{bc} (-i\sigma^{\mu\nu} k'_\nu) u^c(\mathbf{P_\Delta}) \Big\} \overline{u}^d(\mathbf{P_\Delta}) i\gamma_5 u(\mathbf{p_N})$$

$$\Big[\overline{u}(\mathbf{p_N}) \frac{(p'_N + k')_\sigma \gamma^\sigma + m_N}{2m_N} \mu_{N'} < out; \mathbf{p'}_\pi | J(0) | \mathbf{p}_\pi \mathbf{p}_N; in >$$

$$- \mu_N < out; \mathbf{p'}_\pi \mathbf{p'}_N | \overline{J}(0) | \mathbf{p}_\pi; in > \frac{(p_N - k')_\sigma \gamma^\sigma + m_N}{2m_N} u(\mathbf{p'_N}) \Big], \tag{3.8}$$

where $(p'_N . p_N) = (p'_N)_\sigma (p_N)^\sigma$ and in $u^b(\mathbf{P_\Delta})$ $u^c(\mathbf{P'_\Delta})$ the four-momentums P_Δ and P'_Δ with complex mass $\Sigma(s) = M_\Delta(s) - \frac{i\Gamma_\Delta(s)}{2}$ are assumed

$$P^o_\Delta(s) = \sqrt{(M_\Delta(s) - \frac{i\Gamma_\Delta(s)}{2})^2 + \mathbf{P}^2_\Delta}; \qquad \mathbf{P}_\Delta = \mathbf{p}_N + \mathbf{p}_\pi \tag{3.9a}$$

$$P'^o_\Delta(s') = \sqrt{(M_\Delta(s') - \frac{i\Gamma_\Delta(s')}{2})^2 + \mathbf{P'}^2_\Delta}; \qquad \mathbf{P'}_\Delta = \mathbf{p'}_N + \mathbf{p'}_\pi. \tag{3.9b}$$

$\mathcal{H}^\mu_{\gamma'\pi'N'-\pi N}$ has the s-channel two-body poles at $s = s'$ and the Δ resonance poles in the complex energy region from the πN amplitudes. Therefore $\mathcal{H}^\mu_{\gamma'\pi'N'-\pi N}$ can be represented in the form of the double Δ (or πN cluster) exchange term (Fig. 2B) if we use the s-channel pole approximation for the πN amplitudes (2.9a,b,c,d) in the Δ resonance region

$$\overline{u}(\mathbf{p_N})\frac{(p'_N+k')_\sigma\gamma^\sigma + m_N}{2m_N}\overline{u}(\mathbf{p'}_N) < out; \mathbf{p'}_\pi|J(0)|\mathbf{p}_\pi\mathbf{p}_N; in > \Longrightarrow \frac{\mathrm{R}_{N'}}{p^o_\pi + p^o_N - P'^o_\Delta(s)}; \tag{3.10a}$$

$$\overline{u}(\mathbf{p_N})u(\mathbf{p'_N}) < out; \mathbf{p'}_N|j_{\pi'}(0)|\mathbf{p}_\pi\mathbf{p}_N; in > \Longrightarrow \frac{\mathrm{R}_{\pi'}}{p^o_\pi + p^o_N - P^o_\Delta(s)}; \tag{3.10b}$$

$$< out; \mathbf{p'}_\pi\mathbf{p'}_N|\overline{J}(0)|\mathbf{p}_\pi; in > u(\mathbf{p}_N)\frac{(p_N-k')_\sigma\gamma^\sigma + m_N}{2m_N}u(\mathbf{p'_N}) \Longrightarrow \frac{\mathrm{R}_N}{p'^o_\pi + {p'_N}^o - P'^o_\Delta(s')}; \tag{3.10c}$$

$$< out; \mathbf{p'}_\pi\mathbf{p'}_N|j_\pi(0)|\mathbf{p}_N; in > \overline{u}(\mathbf{p_N})u(\mathbf{p'_N}) \Longrightarrow \frac{\mathrm{R}_\pi}{p'^o_\pi + p'^o_N - P'^o_\Delta(s')}, \tag{3.10d}$$

The present form of the Δ propagator in eq. (3.10a,b,c,d) is similar to the quantum mechanical form of the Breit-Wigner propagator, where a imagenery part of energy of resonance is generated by a retardation effect [24].

$\mathcal{D}^\mu_{\gamma'\pi'N'-\pi N}$ (3.7a) satisfies the same current conservation condition (3.3b) as $\widetilde{\mathcal{E}}^\mu_{\gamma'\pi'N'-\pi N}$ (2.15) and $\mathcal{M}^\mu_{\gamma'\pi'N'-\pi N}$ (3.4a). Unlike the $\mathcal{D}^\mu_{\gamma'\pi'N'-\pi N}$ amplitude $\mathcal{H}^\mu_{\gamma'\pi'N'-\pi N}$ contains projections on the intermediate spin 3/2 states. Therefore $\mathcal{H}^\mu_{\gamma'\pi'N'-\pi N}$ satisfies a modified current conservation condition

$$\Big[k'_\mu < out; \mathbf{p'}_N\mathbf{p'}_\pi|J^\mu(0)|\mathbf{p}_\pi\mathbf{p}_N; in >\Big]^{\textit{Projection on spin 3/2 particle states}}_{\textit{on mass shell } \pi',\ N',\ \pi,\ N} = k'_\mu\mathcal{H}^\mu_{\gamma'\pi'N'-\pi N} + b_{\gamma'\pi'N'-\pi N} = 0, \tag{3.11a}$$

where

$$b_{\gamma'\pi'N'-\pi N} = \frac{(p'_N)_a(p_N)_d}{(p'_N.p_N)}\overline{u}(\mathbf{p'_N})i\gamma_5 u^a(\mathbf{P'})\Big\{\overline{u}^b(\mathbf{P'})g_{bc}u^c(\mathbf{P})\Big\}\overline{u}^d(\mathbf{P})i\gamma_5 u(\mathbf{p_N})$$
$$\Big[\overline{u}(\mathbf{p_N})\frac{(p'_N+k')_\sigma\gamma^\sigma + m_N}{2m_N}e_{N'} < out; \mathbf{p'}_\pi|J(0)|\mathbf{p}_\pi\mathbf{p}_N; in >$$

$$-e_N < out; \mathbf{p'}_\pi \mathbf{p'}_N | \overline{J}(0) | \mathbf{p}_\pi; in > \frac{(p_N - k')_\sigma \gamma^\sigma + m_N}{2m_N} u(\mathbf{p'_N})$$

$$+\overline{u}(\mathbf{p_N}) u(\mathbf{p'_N}) e_{\pi'} < out; \mathbf{p'}_N | j_{\pi'}(0) | \mathbf{p}_\pi \mathbf{p}_N; in > -e_\pi < out; \mathbf{p'}_\pi \mathbf{p'}_N | j_\pi(0) | \mathbf{p}_N; in > \overline{u}(\mathbf{p_N}) u(\mathbf{p'_N}) \Big] \tag{3.11b}$$

is obtained from $\widetilde{\mathcal{B}}_{\gamma'\pi'N'-\pi N}$ (2.14) after the intermediate projection on the spin 3/2 particle states.

Substitution of (3.10a,b,c,d) into (3.8) gives

$$\mathcal{H}^\mu_{\gamma'\pi'N'-\pi N} = \frac{1}{(p'_N . p_N)} \overline{u}(\mathbf{p'_N})(p'_N)_a i\gamma_5 u^a(\mathbf{P'})$$

$$\Big\{ \overline{u}^b(\mathbf{P'}) g_{bc} \Big[(P+P')^\mu \mathcal{V}_E - i\sigma^{\mu\nu} k'_\nu \mathcal{V}_H \Big] u^c(\mathbf{P}) \Big\} \overline{u}^d(\mathbf{P})(p_N)_d i\gamma_5 u(\mathbf{p_N}), \tag{3.12a}$$

where

$$\mathcal{V}_E = \frac{e_{N'} \mathrm{R}_{N'}}{(s-s')\Big(p^o_\pi + p^o_N - P^o_\Delta(s)\Big)} + \frac{e_{\pi'} \mathrm{R}_{\pi'}}{(s-s')\Big(p^o_\pi + p^o_N - P^o_\Delta(s)\Big)}$$

$$- \frac{e_N \mathrm{R}_N}{(s-s')\Big(p'^o_\pi + p'^o_N - P'^o_\Delta(s')\Big)} - \frac{e_\pi \mathrm{R}_\pi}{(s-s')\Big(p'^o_\pi + p'^o_N - P'^o_\Delta(s')\Big)}, \tag{3.13a}$$

$$\mathcal{V}_H = \frac{\mu_{N'} \mathrm{R}_{N'}}{(s-s')\Big(p^o_\pi + p^o_N - P^o_\Delta(s)\Big)} - \frac{\mu_N \mathrm{R}_N}{(s-s')\Big(p'^o_\pi + p'^o_N - P'^o_\Delta(s')\Big)}. \tag{3.13b}$$

After a simple algebra (3.13b) can be rewritten as

$$\mathcal{V}_H = \frac{1}{\Big(p'^o_\pi + p'^o_N - P'^o_\Delta(s')\Big)\Big(p^o_\pi + p^o_N - P^o_\Delta(s)\Big)} \Big\{ R_+ \Big[\frac{|\mathbf{k'}|}{s-s'} - \frac{P^o_\Delta(s) - P'^o_\Delta(s')}{s-s'} \Big]$$

$$+ \frac{R_-}{s-s'} \Big[p^o_\pi + {p_N}^o + p'^o_\pi + {p'_N}^o - P^o_\Delta(s) - P'^o_\Delta(s') \Big] \Big\} \tag{3.14a}$$

where

$$R_\pm = \frac{1}{2} \Big[\mu_{N'} \mathrm{R}_{N'} \pm \mu_N \mathrm{R}_N \Big]. \tag{3.14b}$$

The first part in (3.14a) is regular at $|\mathbf{k'}| = 0$, where $s = s'$, because $\Big(P^o_\Delta(s) - P'^o_\Delta(s')\Big)/(s-s')$ is finite at $s = s'$. In the πN c.m. frame by $|\mathbf{k'}| = 0$ this part is in order of $1/(\Gamma_\Delta/2)^2$, where Γ_Δ is the Δ decay width.

The second part of $\mathcal{V}_H$ (3.14a) can describe only one Δ exchange because $p^o_\pi + {p_N}^o + p'^o_\pi + {p'_N}^o - P^o_\Delta(s) - P'^o_\Delta(s')$ cancel with the one of the Δ propagators. This expression has a different behavior in two respects:

1. For the πN bremsstrahlung reactions without charge exchange $\pi^{\pm}p \longrightarrow \gamma\pi^{\pm}p$ or $\pi^{o}p \longrightarrow \gamma\pi^{o}p$ we have $\mu_{N'} = \mu_N$. In this case $R_-/(s-s')$ is finite at threshold $|\mathbf{k}'| = 0$. In the Δ resonance region the second part of (3.14a) is in order of $1/(\Gamma_\Delta/2)$.

2. For the charge exchange reactions $R_-/(s-s')$ can be singular at threshold $|\mathbf{k}'| = 0$. This case needs a special investigation.

In the complex energy region, where $p^{\mu}_{\pi}+p^{\mu}_{N} \Longrightarrow P^{\mu}_{\Delta}$ and $p'^{\mu}_{\pi}+p'^{\mu}_{N} \Longrightarrow P'^{\mu}_{\Delta}$ both parts of (3.14a) contain the same singularities in the propagators $1/\Big(p'^{o}_{\pi} + p'^{o}_{N} - P'^{o}_{\Delta}(s')\Big)\Big(p^{o}_{\pi} + p^{o}_{N} - P^{o}_{\Delta}(s)\Big)$. Therefore it is convenient to introduce the smooth function in the above limits $\mathcal{V}_H$

$$\mathcal{V}_H = \frac{1}{\Big(p'^{o}_{\pi} + p'^{o}_{N} - P'^{o}_{\Delta}(s')\Big)\Big(p^{o}_{\pi} + p^{o}_{N} - P^{o}_{\Delta}(s)\Big)} \mathrm{g}_{\pi' N'-\Delta'}(s',k')\mathrm{V}_H \mathrm{g}_{\Delta-\pi N}(s), \qquad (3.15a)$$

where $\mathrm{g}_{\Delta-\pi N}(s)$ and $\mathrm{g}_{\pi' N'-\Delta'}(s',k')$ are the radial parts of the $\Delta - \pi N$ form factors from Fig. 2B.

The same transformations for $\mathcal{V}_E$ gives

$$\mathcal{V}_E = \frac{1}{\Big(p'^{o}_{\pi} + p'^{o}_{N} - P'^{o}_{\Delta}(s')\Big)\Big(p^{o}_{\pi} + p^{o}_{N} - P^{o}_{\Delta}(s)\Big)} \mathrm{g}_{\pi' N'-\Delta'}(s',k')\mathrm{V}_E \mathrm{g}_{\Delta-\pi N}(s). \qquad (3.15b)$$

Relations (3.15a,b) enables us to represent external particle radiation amplitude (3.12a) in the form of the double Δ particle exchange amplitude (Fig. 2B)

$$\mathcal{H}^{\mu}_{\gamma'\pi' N'-\pi N} = \frac{-1}{(p'_N . p_N)} \frac{< \mathbf{p}'_N, \mathbf{p}'_\pi | g_{\pi' N'-\Delta'} | \mathbf{P}'_\Delta >}{p'^{o}_{\pi} + p'^{o}_{N} - P'^{o}_{\Delta}(s')}$$
$$\Big\{\overline{u}^b(\mathbf{P}') g_{bc}\Big[(P+P')^{\mu}\mathrm{V}_E - i\sigma^{\mu\nu}k'_{\nu}\mathrm{V}_H\Big]u^c(\mathbf{P})\Big\} \frac{< \mathbf{P}_\Delta | g_{\Delta-\pi N} | \mathbf{p}_N, \mathbf{p}_\pi >}{p^{o}_{\pi} + p^{o}_{N} - P^{o}_{\Delta}(s)}, \qquad (3.12b)$$

where the following $\Delta - \pi N$ and $\pi N - \Delta$ vertex functions are introduced[2]

[2]For the on mass shell πN states with $s = s'$ and $k' = 0$ the operator

$$\mathcal{Q}(\mathbf{p}'_N, \mathbf{p}_N, \mathbf{P}'_\Delta, \mathbf{P}_\Delta) = \frac{1}{(p'_N . p_N)} \overline{u}(\mathbf{p}'_N) i\gamma_5 (p'_N)_a u^a(\mathbf{P}'_\Delta)\overline{u}^d(\mathbf{P}_\Delta)(p_N)_d i\gamma_5 u(\mathbf{p}_N)$$

transforms into the projection operator $\mathcal{P}^{3/2}_1(\mathbf{p}'_N, \mathbf{p}_N)$ which projects on the πN state with the orbital momentum $L = 1$ and the total momentum $J = 3/2$ [18]

$$\mathcal{P}^{3/2}_1(\mathbf{p}'_N, \mathbf{p}_N) = \frac{6m_N}{4\pi \mathbf{p}\mathbf{p}'(m_N + \sqrt{m^2_N + \mathbf{p}^2})}) \overline{u}(\mathbf{p}'_N) i\gamma_5 p'_{N\,a} u^a(\mathbf{P}'_\Delta)\overline{u}^d(\mathbf{P}_\Delta) p_{N\,d} i\gamma_5 u(\mathbf{p}_N),$$

i.e.

$$\Bigg[\mathcal{Q}(\mathbf{p}'_N, \mathbf{p}_N, \mathbf{P}'_\Delta, \mathbf{P}_\Delta)\Bigg]^{|\mathbf{k}'|=0} = \frac{4\pi \mathbf{p}\mathbf{p}'(m_N + \sqrt{m^2_N + \mathbf{p}^2})}{6m_N(p'_N . p_N)} \mathcal{P}^{3/2}_1(\mathbf{p}'_N, \mathbf{p}_N)$$

$$< \mathbf{p'}_N, \mathbf{p'}_\pi | g_{\pi' N' - \Delta} | \mathbf{P'}_\Delta >= \mathrm{g}_{\pi' N' - \Delta'}(s', k') \overline{u}(\mathbf{p'}_N) i\gamma_5 p'_{N a} u^a(\mathbf{P'}_\Delta), \tag{3.16a}$$

$$< \mathbf{P}_\Delta | g_{\Delta - \pi N} | \mathbf{p}_N, \mathbf{p}_\pi >= \mathrm{g}_{\Delta - \pi N}(s) \overline{u}^d(\mathbf{P}_\Delta) p_{N d} i\gamma_5 u(\mathbf{p}_N). \tag{3.16b}$$

The total πN bremsstrahlung amplitude $< out; \mathbf{p'}_N \mathbf{p'}_\pi | J^\mu(0) | \mathbf{p}_\pi \mathbf{p}_N; in >$ is a sum of the photon radiation diagrams from the external particles (2.8b) (Fig. 1) and the infinite set of other amplitudes with the photon radiation diagrams from the internal particles $\mathcal{I}^\mu_{\gamma' \pi' N' - \pi N}$. A symbolical picture of this internal amplitude is given on Fig. 2A.

$$< out; \mathbf{p'}_N \mathbf{p'}_\pi | J^\mu(0) | \mathbf{p}_\pi \mathbf{p}_N; in >= \mathcal{E}^\mu_{\gamma' \pi' N' - \pi N} + \mathcal{I}^\mu_{\gamma' \pi' N' - \pi N}. \tag{3.17a}$$

The current conservation allows to use the well known Low prescription [1] for an estimation of $\mathcal{I}^\mu_{\gamma' \pi' N' - \pi N}$ on the basis of the external particle radiation amplitude. In particular, from the current conservation (3.11a) follows the existence of an internal particle radiation amplitude $\mathrm{I}^\mu_{\gamma' \pi' N' - \pi N}$ which satisfies the condition

$$k'_\mu \widetilde{\mathcal{I}}^\mu_{\gamma' \pi' N' - \pi N} = \mathrm{b}_{\gamma' \pi' N' - \pi N}, \tag{3.17b}$$

where the intermediate antinucleon degrees of freedom are omitted.

From the infinite set of the internal particle radiation diagrams we take the double Δ exchange term $\mathrm{H}^\mu_{\gamma' \pi' N' - \pi N}$ [3] (Fig. 2B) which has the same analytical properties as $\mathcal{H}^\mu_{\gamma' \pi' N' - \pi N}$ (3.12b). This term contains the full $\Delta - \gamma' \Delta'$ vertex function $< \mathbf{P'}_\Delta | J_\mu(0) | \mathbf{P}_\Delta >$.

$$\mathrm{H}^\mu_{\gamma' \pi' N' - \pi N} = \frac{-1}{(p'_N . p_N)} \frac{< \mathbf{p'}_N, \mathbf{p'}_\pi | g_{\pi' N' - \Delta'} | \mathbf{P'}_\Delta >}{p'^o_\pi + p'^o_N - P'^o_\Delta(s')} < \mathbf{P'}_\Delta | J^\mu(0) | \mathbf{P}_\Delta > \frac{< \mathbf{P}_\Delta | g_{\Delta - \pi N} | \mathbf{p}_N, \mathbf{p}_\pi >}{p^o_\pi + p^o_N - P^o_\Delta(s)}. \tag{3.18a}$$

This expression can be exactly extracted from the πN bremsstrahlung amplitude as it was done for the amplitude of the $\gamma p \to \gamma' \pi' N'$ reaction in our previous papers [17, 18]. Corresponding recipe is given in appendix B. A connection between the external particle radiation diagrams on Fig. 1 and double Δ exchange term on Fig. 2 in tree approximation are considered in [7] using the Brodsky-Brown decomposition identities [14, 15].

The general form of the $\Delta - \gamma' \Delta'$ vertex is

$$< \mathbf{P'}_\Delta | J^\mu(0) | \mathbf{P}_\Delta >= (P_\Delta + P'_\Delta)^\mu \Big(\overline{u}^\sigma(\mathbf{P'}_\Delta) \Big[g_{\rho\sigma} G_1(k'^2_\Delta) + k'_{\Delta\sigma} k'_{\Delta\rho} G_3(k'^2_\Delta) \Big] \Big) u^\rho(\mathbf{P}_\Delta)$$

$$+ \overline{u}^\sigma(\mathbf{P'}_\Delta) \Big(-i\sigma^{\mu\nu} k'_{\Delta\nu} \Big[g_{\rho\sigma} G_2(k'^2_\Delta) + k'_{\Delta\sigma} k'_{\Delta\rho} G_4(k'^2_\Delta) \Big] \Big) u^\rho(\mathbf{P}_\Delta), \tag{3.19a}$$

[3]An other double Δ exchange term contains the $\Delta - \pi' \Delta'$ vertex function. This term has a different analytical behavior as (3.12b) and (3.18a) with $\Delta - \gamma' \Delta'$ vertex function. In addition, the term with $\Delta - \pi' \Delta'$ vertex is small [3].

where $k'_{\Delta\mu} = (P_\Delta - P'_\Delta)_\mu$, $g_{\mu\nu}$ is the metric tensor and the form factors $G_i(k'^2_\Delta)$ are simply connected with the charge monopole $G_{C0}(k'^2_\Delta)$, the magnetic dipole $G_{M1}(k'^2_\Delta)$, the electric quadrapole $G_{E2}(k'^2_\Delta)$ and the magnetic octupole $G_{M3}(k'^2_\Delta)$ form factors of the Δ resonance. [4] In the low energy region we can neglect the terms $\sim k'^2_\Delta/4M^2_\Delta$, and keep only terms $\sim 1/M_\Delta$. Then the previous formula can be rewritten in a similar form as the γNN vertex function:

$$< \mathbf{P'}_\Delta|J_\mu(0)|\mathbf{P}_\Delta >= \overline{u}^\sigma(\mathbf{P'}_\Delta) g_{\rho\sigma}\Big[\frac{(P_\Delta + P'_\Delta)_\mu}{2M_\Delta} G_{C0}(k'^2_\Delta) - \frac{i\sigma_{\mu\nu}k'^\nu_\Delta}{2M_\Delta} G_{M1}(k'^2_\Delta)\Big] u^\rho(\mathbf{P}_\Delta) \qquad (3.19b)$$

The form of the expressions (3.19a,b) insures the validity of the the one-body current conservation in the complex Δ-resonance energy region

$$k'^\mu_\Delta < \mathbf{P'}_\Delta|J_\mu(0)|\mathbf{P}_\Delta >= 0, \quad k'_{\Delta\mu}\mathrm{H}^\mu_{\gamma'\pi'N'-\pi N} = 0. \qquad (3.18b)$$

Thus after projections on the spin 3/2 states in the full πN bremsstrahlung amplitude $< out; \mathbf{p'}_N \mathbf{p'}_\pi|J^\mu(0)|\mathbf{p}_\pi \mathbf{p}_N; in >$ (3.17a) we have separated two leading amplitudes (3.12b) and (3.18a) with the double Δ-exchange poles and $\Delta - \Delta'\gamma'$ vertex. Unfortunately $\mathcal{H}^\mu_{\gamma'\pi'N'-\pi N}$ (3.12b) contains also a contributions from the one Δ exchange (see consideration after eq. (3.14b)). This one can see from the structure of $\mathcal{H}^\mu_{\gamma'\pi'N'-\pi N}$ which does not satisfy the one body current conservation condition $k'_{\Delta\mu}\mathcal{H}^\mu_{\gamma'\pi'N'-\pi N} \neq 0$. Therefore in order to compare $\mathcal{H}^\mu_{\gamma'\pi'N'-\pi N}$ with $\mathrm{H}^\mu_{\gamma'\pi'N'-\pi N}$ we must modify $\mathcal{H}^\mu_{\gamma'\pi'N'-\pi N}$ as

$$\mathcal{H}^\mu_{\gamma'\pi'N'-\pi N} \longrightarrow \widetilde{\mathcal{H}}^\mu_{\gamma'\pi'N'-\pi N}, \quad \widetilde{\mathcal{H}}^\mu_{\gamma'\pi'N'-\pi N} = \mathcal{H}^\mu_{\gamma'\pi'N'-\pi N} - h^\mu_{\gamma'\pi'N'-\pi N}, \qquad (3.20)$$

where

$$\widetilde{\mathcal{H}}^\mu_{\gamma'\pi'N'-\pi N} = \frac{-1}{(p'_N.p_N)} \frac{< \mathbf{p'}_N, \mathbf{p'}_\pi|g_{\pi'N'-\Delta'}|\mathbf{P'}_\Delta >}{p'^o_\pi + p'^o_N - P'^o_\Delta(s')}$$

$$\Big\{\overline{u}^b(\mathbf{P'}) g_{bc}\Big[(P_\Delta + P'_\Delta)^\mu \mathrm{V}_E - i\sigma^{\mu\nu}k'_{\Delta\nu}\mathrm{V}_H\Big]u^c(\mathbf{P})\Big\} \frac{< \mathbf{P}_\Delta|g_{\Delta-\pi N}|\mathbf{p}_N, \mathbf{p}_\pi >}{p^o_\pi + p^o_N - P^o_\Delta(s)}, \qquad (3.21a)$$

$$h^\mu_{\gamma'\pi'N'-\pi N} = \frac{-1}{(p'_N.p_N)} \frac{< \mathbf{p'}_N, \mathbf{p'}_\pi|g_{\pi'N'-\Delta'}|\mathbf{P'}_\Delta >}{p'^o_\pi + p'^o_N - P'^o_\Delta(s')}$$

$$\Big\{\overline{u}^b(\mathbf{P'}) g_{bc}\Big[(P_\Delta + P'_\Delta - P - P')^\mu \mathrm{V}_E - i\sigma^{\mu\nu}(k'_\Delta - k')_\nu\mathrm{V}_H\Big]u^c(\mathbf{P})\Big\} \frac{< \mathbf{P}_\Delta|g_{\Delta-\pi N}|\mathbf{p}_N, \mathbf{p}_\pi >}{p^o_\pi + p^o_N - P^o_\Delta(s)}, \qquad (3.21b)$$

Expression (3.21b) contains factors $(P_\Delta + P'_\Delta - P - P')$ and $k'_\Delta - k'$ which cancel with one of the Δ propagators $1/(P'^o - P'^o_\Delta)$ or $1/(P^o - P^o_\Delta)$. Therefore $h^\mu_{\gamma'\pi'N'-\pi N}$ contributes to the one Δ exchange part of external particle radiation amplitude. This contribution is in order of $2/\Gamma_\Delta$ unlike to the double Δ exchange terms $\sim 4/\Gamma^2_\Delta$. Therefore if we

[4] Other choices of F_i form factors are considered in ref. [11]

keep only the double Δ exchange terms, then $\widetilde{\mathcal{H}}^{\mu}_{\gamma'\pi'N'-\pi N}$ satisfies the one-body current conservation condition

$$
{k'}_{\Delta\mu}\widetilde{\mathcal{H}}^{\mu}_{\gamma'\pi'N'-\pi N}=0 \tag{3.22}
$$

and instead of (3.17b) we get the following real photon current conservation condition

$$
{k'}_{\mu}\Big(\widetilde{\mathcal{H}}^{\mu}_{\gamma'\pi'N'-\pi N}+\mathrm{H}^{\mu}_{\gamma'\pi'N'-\pi N}\Big)=0. \tag{3.23}
$$

It is evident, that relation (3.23) implies a appropriate modification of (3.17b)

$$
k'_{\mu}\widetilde{\mathcal{H}}^{\mu}_{\gamma'\pi'N'-\pi N}=\widetilde{\mathrm{b}}_{\gamma'\pi'N'-\pi N}, \tag{3.17c}
$$

where instead of (3.11b) we get

$$
\widetilde{\mathrm{B}}^{\mu}_{\gamma'\pi'N'-\pi N}=\frac{-1}{(p'_N.p_N)}\frac{<\mathbf{p'}_N,\mathbf{p'}_\pi|g_{\pi'N'-\Delta'}|\mathbf{P'}_\Delta>}{p'^o_\pi+p'^o_N-P'^o_\Delta(s')}
$$

$$
\Big\{\overline{u}^b(\mathbf{P'})g_{bc}\Big[(P_\Delta+P'_\Delta)^\mu\mathrm{V}_E-i\sigma^{\mu\nu}k'_{\Delta\nu}\mathrm{V}_H\Big]u^c(\mathbf{P})\Big\}\frac{<\mathbf{P}_\Delta|g_{\Delta-\pi N}|\mathbf{p}_N,\mathbf{p}_\pi>}{p^o_\pi+p^o_N-P^o_\Delta(s)}, \tag{3.11c}
$$

The importance of eq. (3.23) follows from the cancellation of the internal $\mathrm{H}^{\mu}_{\gamma'\pi'N'-\pi N}$ and external $\widetilde{\mathcal{H}}^{\mu}_{\gamma'\pi'N'-\pi N}$ amplitudes according to current conservation. Thus according to (3.23) a comparison of the $\Delta-\gamma'\Delta'$ vertex functions in (3.21a) and (3.18a) gives

$$
\overline{u}^\sigma(\mathbf{P'}_\Delta)g_{\rho\sigma}\Big[\frac{k'_\mu(P_\Delta+P'_\Delta)^\mu}{2M_\Delta}(2M_\Delta\mathrm{V}_E)-\frac{ik'_\mu\sigma_{\mu\nu}{k'}^\nu_\Delta}{2M_\Delta}(2M_\Delta\mathrm{V}_H)\Big]u^\rho(\mathbf{P}_\Delta)
$$

$$
=-\overline{u}^\sigma(\mathbf{P'}_\Delta)g_{\rho\sigma}\Big[\frac{k'_\mu(P_\Delta+P'_\Delta)^\mu}{2M_\Delta}G_{C0}({k'}^2_\Delta)-\frac{ik'_\mu\sigma_{\mu\nu}{k'}^\nu_\Delta}{2M_\Delta}G_{M1}({k'}^2_\Delta)\Big]u^\rho(\mathbf{P}_\Delta), \tag{3.24}
$$

where we have used a low energy photon approximation which enables us to ignore the electric quadrupole and the magnetic octupole parts of the $\Delta-\gamma\Delta$ vertex functions. Here it's important to note, that $2M_\Delta$ is a natural unit for V_E and V_H, where $2M_\Delta/(s-s')$ can be replaced in Δ resonance region by the linear propagator $\sim 1/(s^{1/2}-s'^{1/2})$ which is used in the considered time-ordered formulation.

Next from (3.24) we get

$$
G_{C0}({k'}^2_\Delta)=-2M_\Delta\mathrm{V}_E;\quad G_{M1}({k'}^2_\Delta)=-2M_\Delta\mathrm{V}_H, \tag{3.25}
$$

where V_E and V_H, as well as $\mathrm{R}_{N'}$, $\mathrm{R}_{\pi'}$, R_N and R_π in (3.10a,b,c,d) are given in the limits

$$
p^\mu_\pi+p^\mu_N\Longrightarrow P^\mu_\Delta,\quad and\quad p'^\mu_\pi+p'^\mu_N\Longrightarrow P'^\mu_\Delta. \tag{3.26a}
$$

Here a special on mass shell momenta of π mesons ($q_\pi=(\sqrt{\mathbf{q}^2_\pi+m^2_\pi},\mathbf{q}_\pi)$, ${q'}_\pi=(\sqrt{\mathbf{q'}^2_\pi+m^2_\pi},\mathbf{q'}_\pi)$) and of nucleons ($q_N=(\sqrt{\mathbf{q}^2_N+m^2_N},\mathbf{q}_N)$, ${q'}_N=(\sqrt{\mathbf{q'}^2_N+m^2_N},\mathbf{q'}_N)$ are introduced. These four-momenta have the equal radial part and the same direction as the corresponding on-mass shell four momenta $\mathbf{p}$

$$\mathbf{q}_N = q\frac{\mathbf{p}_N}{p_N}, \quad \mathbf{q}_\pi = q\frac{\mathbf{p}_\pi}{p_\pi}, \quad \mathbf{q}'_N = q\frac{\mathbf{p}'_N}{p'_N}, \quad \mathbf{q}'_\pi = q\frac{\mathbf{p}'_\pi}{p'_\pi}, \tag{3.26b}$$

where

$$q^2 = \frac{\left(m_\Delta^2 - (m_N + m_\pi)^2\right)\left(m_\Delta^2 - (m_N - m_\pi)^2\right)}{4m_\Delta^2}. \tag{3.26c}$$

In V_E and V_H integrations are implied over the relative π-N angles due to projection on the states with the spin 3/2 and $L = 1$ orbital momenta. Finally V_E and V_H are dependent only on k'^2_Δ. It must be emphasized, that the Ward-Takahashi identity (2.4) remains to be valid for the bremsstrahlung amplitude in the limits $\lim_{p^o_\pi + p^o_N \Longrightarrow P^o_\Delta} \lim_{p'^o_\pi + p'^o_N \Longrightarrow P'^o_\Delta} < out; \mathbf{p}'_N \mathbf{p}'_\pi | J^\mu(0) | \mathbf{p}_\pi \mathbf{p}_N; in >$, because q_N, q_π, q'_N and q'_π are on mass shell.

According to (3.25) the electric and magnetic parts of the internal and external particle radiation amplitudes with double Δ exchange and $\Delta - \Delta\gamma$ vertex cancel each other. Therefore the full πN bremsstrahlung amplitude become independent on these amplitudes. Thus the role of the double Δ exchange amplitude reduces to decrease the external particle radiation parts of the πN bremsstrahlung amplitude. Consequently we have screening of the internal Δ radiation part by the external particle radiation diagrams.

Equation (3.25) indicates that for V_E (3.15a) a normalization condition is fulfilled for V_E and G_{C0}

$$G_{C0}(0) = -\Big[2M_\Delta \mathrm{V}_E\Big]^{k'=0} = e_\Delta \tag{3.27}$$

where e_Δ denotes the charge of Δ.

The normalization condition (3.27) can be used also for V_H in the special cases for neutral pion reactions $\pi^o p \to \gamma' \pi^{o\prime} p'$ and $\gamma p \to \gamma' \pi^o p'$, where V_E and V_H contains only nucleon parts and they are determined by the same expression in the square brackets of eq. (3.8). Then according to (3.27) it is easy get

$$\mu_{\Delta^+} = G_{M1}(0) = -\Big[2M_\Delta \mathrm{V}_H\Big]^{k'=0} = \mu_p \frac{M_\Delta}{m_N} \tag{3.28}$$

Magnetic momenta of Δ^{++} can be obtained from (3.28) using the isotopic symmetry between the $\pi^o p \to \pi^o p$ and $\pi^+ p \to \pi^+ p$ amplitudes. Then we get $\mu_{\Delta^{++}} = \frac{3}{2}\mu_{\Delta^+}$. Unfortunately one can not use the $\pi^o n \to \gamma' \pi^{o\prime} n'$ reaction for determination of the magnetic moments of Δ^o and Δ^-, because equal-time commutators (2.3a,b) are zero in that case.

The present procedure of construction of the bremsstrahlung amplitude does not contradict to the quantum-electro-dynamical (QED) renormalisation recipe of the charge and the mass. Thus if we assume that the perturbation series is convergent, then we can operate with the physical charge and the physical mass which enter in the above relation for the equal time commutators and in the corresponding amplitudes. Moreover, the renormalisation of the already renormalized expressions generates additional conditions [16] which preserve the form of the equal-time commutators and other principal conditions.

4. Conclusion

As basis for the study of the πN bremsstrahlung we have used a two-body form of the Ward-Takahashi identities (2.4) which generates following model-independent relations:

(**i**) An amplitude of an arbitrary $a+b \longrightarrow \gamma' + f_1 + ... + f_n$ ($n = 1, 2, ...$) reaction fulfill generalized current conservations

$$k'_\mu < out; f_1, ..., f_n | J^\mu(0) | a, b; in >= \Big[\mathcal{B}_{\gamma' f_1 ... f_n - ab} + k'_\mu \mathcal{E}^\mu_{\gamma' f_1 .. f_n - ab}\Big]_{on\ mass\ shell\ f_1,\ .\ .\ .\ f_n;\ a,\ b} = 0, \tag{I}$$

where $\mathcal{E}^\mu_{\gamma' f_1 .. f_n - ab}$ corresponds to the complete set of Feynman (or three-dimensional time-ordered) diagrams with the photo-emission from each external particles and

$$\mathcal{B}_{\gamma' f_1 .. f_n - ab} = \sum_{m=1 (I_1 \neq m ... I_{n-1} \neq m)}^{n} e_m < out; f_{I_1} ... f_{I_{n-1}} | J_m(0) | a, b; in >$$
$$- e_a < out; f_1 ... f_n | J_a(0) | b; in > - e_b < out; f_1 ... f_n | J_b(0) | a; in > \tag{II}$$

stands for amplitudes of the $a + b \longrightarrow f_1 + ... + f_n$ reaction without γ' emission.

A special case of relation (I) is given by eq. (2.7a,b) with the external particle radiation diagrams on Fig. 1.

Equation (I) and (II) are also valid for an arbitrary number of external photons. For instance, these equations can be used as the current conservation conditions for the pion photo-production reaction $\gamma A \rightarrow \pi' A'$, for Compton scattering $\gamma A \rightarrow \gamma' A'$ etc. Moreover the current conservation condition (I) can be extended in the off mass shell region as it is done in eq. (2.7b).

(**ii**) The current conservation (I) requires the existence of the internal particle radiation amplitude $\mathcal{I}^\mu_{\gamma' f_1 .. f_n - ab}$ which ensures the validity of this condition

$$k'_\mu \mathcal{I}^\mu_{\gamma' f_1 .. f_n - ab} = \mathcal{B}_{\gamma' f_1 ... f_n - ab}, \ \ or\ \ k'_\mu \mathcal{E}^\mu_{\gamma' f_1 .. f_n - ab} + k'_\mu \mathcal{I}^\mu_{\gamma' f_1 .. f_n - ab} = 0. \tag{III}$$

This means that $\mathcal{E}^\mu_{\gamma' f_1 .. f_n - ab}$ and $\mathcal{I}^\mu_{\gamma' f_1 .. f_n - ab}$ have a different sign and they must be subtracted from each other. Thus we have a screening of the internal particle radiation amplitudes by the external one-particle radiation terms.

(**iii**) For the soft emitted photons $k' \rightarrow 0$ our approach immediately reproduces the low energy theorems for the bremsstrahlung reactions.

(**iv**) The external particle radiation part of the bremsstrahlung amplitude $\mathcal{E}^\mu$ contains the electromagnetic form factors of the external particles only in the tree approximation. This follows from the equal-time commutators (2.3a,b) which are a result of the charge conservation. Thus we must modify the equal-time commutators (2.3a,b) between the Heisenberg operators of the external particles in order to apply the full electromagnetic form factors of pions and nucleons in the current conservation condition (I) or (III).

The above screening mechanism has been applied to the πN bremsstrahlung reaction with the leading double Δ exchange term (Fig. 2B). We have shown, that in the low

energy region, where the electric quadruple and the magnetic octupole momenta of Δ can be neglected, the above double Δ exchange term is completely canceled with the corresponding part of the external particle radiation amplitude. From this cancellation follows the normalization condition (3.25) for the Coulomb monopole part of the $\Delta-\gamma'\Delta'$ vertex which allows us to extract the Δ^+ and Δ^{++} dipole magnetic momenta $\mu_{\Delta^+} = G_{M1}(0) = \frac{M_\Delta}{m_N}\mu_p$ and $\mu_{\Delta^{++}} = \frac{3}{2}\mu_{\Delta^+} = 5.46e/2m_p$ or $\mu_{\Delta^{++}}/\mu_p \sim 1.95$. Our result for $\mu_{\Delta^{++}}$, based on the model independent current conservation condition, is in agreement with the prediction of the naive $SU(6)$ quark model for $\mu_{\Delta^{++}} = 2\mu_p = 5.58e/2m_p$ [25, 26], with the nonrelativistic potential model [30] $\mu_{\Delta^{++}} = 4.6 \pm 0.3$. and with extraction of $\mu_{\Delta^{++}}$ from the $\pi^+p \to \gamma\pi^+p$ experimental cross section in the framework of the low energy photon approach $\mu_{\Delta^{++}} = 3.6 \pm 2.0$ [4], $\mu_{\Delta^{++}} = 5.6 \pm 2.1$ [5] and $\mu_{\Delta^{++}} = 4.7 - 6.9$[28]. Our result is larger as the predictions in the modified $SU(6)$ models [31, 27] and in the soft-photon approximation $\mu_{\Delta^{++}} = 3.7 \sim 4.9e/2m_p$ [8]. On the other hand our result is smaller as the values obtained in the framework of the effective meson-nucleon Lagrangian $\mu_{\Delta^{++}} = 6.1 \pm 0.5e/2m_p$ [34], in the effective quark model $\mu_{\Delta^{++}} = 6.17e/2m_p$ [35] and in the modified bag model $\mu_{\Delta^{++}} = 6.54$ [32].

The summary of the numerical estimations of the magnetic moments of Δ^+ and Δ^{++} resonances is given in table 1. In a number of approaches the magnetic moment of Δ is treated as an adjustable parameter in the radiative πN scattering which is determined using the most sensitive configurations to the $\Delta - \gamma\Delta$ vertex in the slow photon regime. Corresponding results obtained from the experimental cross sections of the $\pi^+p \to \gamma\pi^+p$ reaction are indicated in the table 1 with the index f. It must be emphasized, that only our approach and naive $SU(6)$ quark model gives an analytical form for μ_{Δ^+} and $\mu_{\Delta^{++}}$. But our result for μ_{Δ^+} is $M_\Delta/m_p \sim 1.31$-times larger as $\mu_{\Delta^+} = \mu_p = 2.79e/2m_p$ in refs. [25, 35].

Table 1

Magnetic moments of Δ^+ and Δ^{++} in units of the nuclear magneton $\mu_N = e/2m_N$. The ref. in front of the index f indicates the theoretical model which is used to fit of the experimental data and to extract the magnetic moment μ_Δ.

MODELS	*This work*	$SU(6)$	*Potential and K-matrix appr.*	*Modified Bag*	*Soft photon theorem*	*Eff. πN Lagran.*	*Eff. quark*
μ_{Δ^+}	3.64	2.79 [25, 26]					2.79[35]
$\mu_{\Delta^{++}}$	5.46	5.58 [25, 26] 4.25[27] 4.41-4.89[31]	6.9-9.7[29]f 4.6±0.3[30]f 5.6-7.5[33]f	6.54[32]	3.6±2.0[4]f 5.6±2.1[5]f 4.7-6.9[28]f 3.7-4.9[6]f	6.1±0.5[34]f	6.17[35]

This screening mechanism can be observed in the cross sections of the πN bremsstrahlung reaction or in the $\gamma p \to \gamma\pi^o p$ reaction by comparison of the cross sections in and outside the Δ resonance region. Due to the importance of the double Δ exchange diagram

(Fig. 2B) one must have a different $1/k'$ behavior of the bremsstrahlung amplitude in and outside the Δ resonance region.

Our approach is based on current conservation in the usual local quantum theory[12, 13]. This approach is not dependent on the form of the Lagrangians or on the choice of the model of the πN amplitudes i.e. it is model independent. But this formulation does not include the quark degrees of freedom which violate the basic equal-time commutators (2.3a,b) for the Ward-Takahashi identity. Therefore deviation of our result from the experimental magnetic moments of Δ-s can indicate the role of quark degrees of freedom in the electromagnetic interaction of Δ.

In classical electrodynamic it is known, that the current generated by acceleration of the charge e is $J_\mu = ie\Big(p'_\mu/(k'p') - p_\mu/(k'p)\Big)$. Consequently by acceleration of two charged objects e_1 and e_2 we get a current $J^\mu = ie\Big(p_1'^{\,\mu}/(k'p_1')V(1'.2') + p_2'^{\,\mu}/(k'p_2')V(2'.1') - p_1^\mu/(k'p_2)V(1,2) - p_2^\mu/(k'p_2)V(2,1)\Big)$, where $V(1,2)$ indicates a interaction between particle 1 and 2. Thus in classical electrodynamic the form of the currents is the same as in quantum field theory. Therefore the external particle radiation amplitude (2.8b) and (2.15) can be interpreted as the sum of external pion and nucleon one-body currents with the quantum mechanical πN corrections. One can hope, that the screening mechanism works also in classical electrodynamic.

Appendix A: Projections on the intermediate spin 3/2 states

Projection operator on the spin 3/2 state with mass $s^{1/2}$ and four-momenta $P = \left(\sqrt{s+\mathbf{P}^2}, \mathbf{P}\right)$ is [19, 20, 21, 22]

$$\Lambda^{a,b}(\mathbf{P}) = \sum_{S=-3/2}^{3/2} u^a(\mathbf{P},S)\overline{u}^b(\mathbf{P},S) = \frac{\gamma_\sigma P^\sigma + s^{1/2}}{2s^{1/2}}\left[P^{3/2}\right]^{ab} \qquad (A.1a)$$

where

$$\left[P^{3/2}\right]^{ab} = g^{ab} - \frac{1}{3}\gamma^a\gamma^b - \frac{1}{3s}(\gamma_\sigma P^\sigma \gamma^a P^b + \gamma^b P^a \gamma_\sigma P^\sigma) \qquad (A.2a)$$

These expressions were derived [23, 20] imposing a restriction $\gamma_a \left[P^{3/2}\right]^{ab} = 0$ and using the commutation condition of $\left[P^{3/2}\right]^{ab}$ and $(\gamma_\sigma P^\sigma + s^{1/2})/(2s^{1/2})$ projection operators. In consequence of these conditions one obtains a result an identical with (A.1a)

$$\Lambda^{a,b}(\mathbf{P}) = \left[P^{3/2}\right]^{ab}\frac{\gamma_\sigma P^\sigma + s^{1/2}}{2s^{1/2}}. \qquad (A.1b)$$

On mass shell $\left[P^{3/2}\right]^{ab}$ is equal to a well known expression [20]

$$\left[P^{3/2}\right]^{ab} = g^{ab} - \frac{1}{3}\gamma^a\gamma^b - \frac{2P^aP^b}{3s} + \frac{\gamma^aP^b - \gamma^bP^a}{3s^{1/2}} \qquad (A.2b)$$

Operator $\left[P^{3/2}\right]^{ab}$ (A.2a,b) together with spin 1/2 projection operators $\left[P^{1/2}\right]^{ab}$ satisfy a completeness condition

$$\left[P^{3/2}\right]^{ab} + \left[P^{1/2}_{11}\right]^{ab} + \left[P^{1/2}_{22}\right]^{ab} = g^{ab}, \qquad (A.3)$$

where

$$\left[P^{1/2}_{11}\right]^{ab} = \frac{1}{3}\gamma^a\gamma^b - \frac{2}{3s}P^aP^b + \frac{1}{3s}(\gamma_\sigma P^\sigma\gamma^a P^b + \gamma^b P^a\gamma_\sigma P^\sigma), \qquad (A.4a)$$

$$\left[P^{1/2}_{22}\right]^{ab} = \frac{2}{3s}P^aP^b \qquad (A.4b)$$

Afterwards the completeness condition for the projection operators of spin 3/2 particles takes a form

$$\sum_{S=-3/2}^{3/2}\left(u^a(\mathbf{P},S)\overline{u}^b(\mathbf{P},S) + \frac{\gamma_\sigma P^\sigma + s^{1/2}}{2s^{1/2}}\left\{\left[P^{1/2}_{11}\right]^{ab} + \left[P^{1/2}_{22}\right]^{ab}\right\}\right) +$$

$$\sum_{S=-3/2}^{3/2}\left(v^a(\mathbf{P},S)\overline{v}^b(\mathbf{P},S)+\frac{-\gamma_\sigma P^\sigma+s^{1/2}}{2s^{1/2}}\left\{\left[P^{1/2}_{11}\right]^{ab}+\left[P^{1/2}_{22}\right]^{ab}\right\}^*\right)=g^{ab} \qquad (A.5)$$

where $v^a(\mathbf{P},S)$ is bispinor of antiparticle with spin 3/2.

Now one can decompose the $\gamma N-N$ vertex function over the intermediate spin 3/2 states

$$g_{bc}\overline{u}(\mathbf{p'_N})\gamma_5 g^{ab}\Big[e_{N'}(P+P')^\mu-i\mu_{N'}\sigma^{\mu\nu}k'_\nu\Big]g^{cd}\gamma_5 u(\mathbf{p'_N}+\mathbf{k'})=$$

$$\overline{u}(\mathbf{p'_N})\gamma_5\sum_{S=-3/2}^{3/2}\Big(u^a(\mathbf{P},S)\overline{u}^b(\mathbf{P},S)+v^a(\mathbf{P},S)\overline{v}^b(\mathbf{P},S)+...\Big)g_{bc}\Big[e_{N'}(P+P')^\mu-i\mu_{N'}\sigma^{\mu\nu}k'_\nu\Big]$$

$$\sum_{S'=-3/2}^{3/2}\Big(u^c(\mathbf{P},S)\overline{u}^d(\mathbf{P},S)+v^c(\mathbf{P},S)\overline{v}^d(\mathbf{P},S)+...\Big)\gamma_5 u(\mathbf{p'_N}+\mathbf{k'}) \qquad (A.6a)$$

In the Δ-resonance region one can take into account only spin 3/2 intermediate states. Then without antiparticle degrees of freedom and without spin 1/2 intermediate states we get

$$\Big[(p'_N)_a(p_N)^a\overline{u}(\mathbf{p'_N})\gamma_5\Big[e_{N'}(P+P')^\mu-i\mu_{N'}\sigma^{\mu\nu}k'_\nu\Big]\gamma_5 u(\mathbf{p'_N}+\mathbf{k'})\Big]^{Projection\ on\ spin\ 3/2\ states}=$$

$$\sum_{S,S'=-3/2}^{3/2}\overline{u}(\mathbf{p'_N})\gamma_5 {p'_N}_a u^a(\mathbf{P'},S')\Big\{\overline{u}^b(\mathbf{P'},S')g_{bc}\Big[e_{N'}(P+P')^\mu$$

$$-i\mu_{N'}\sigma^{\mu\nu}k'_\nu\Big]u^c(\mathbf{P},S)\Big\}\overline{u}^d(\mathbf{P},S)(p_N)_d\gamma_5 u(\mathbf{p'_N}+\mathbf{k'}). \qquad (A.6b)$$

The completeness condition (A.6a) with the spin 3/2 states only enables us to represent the πN amplitude in the convenient form

$$<out;\mathbf{p'}_N|j_{\pi'}(0)|\mathbf{p}_\pi\mathbf{p}_N;in>\Longrightarrow\overline{u}(\mathbf{p'}_N)\gamma_5^2 u(\mathbf{p}_N)\overline{u}(\mathbf{p}_N)u(\mathbf{p'}_N)<out;\mathbf{p'}_N|j_{\pi'}(0)|\mathbf{p}_\pi\mathbf{p}_N;in>\Longrightarrow\frac{1}{(p'_N.p_N)}$$

$$\Big[\overline{u}(\mathbf{p'}_N)\gamma_5(p'_N)_a u^a(\mathbf{P'})\Big\{\overline{u}^b(\mathbf{P'})g_{bc}u^c(\mathbf{P})\Big\}\overline{u}^d(\mathbf{P})(p_N)_d\gamma_5 u(\mathbf{p}_N)\Big]\overline{u}(\mathbf{p}_N)u(\mathbf{p'}_N)<out;\mathbf{p'}_N|j_{\pi'}(0)|\mathbf{p}_\pi\mathbf{p}_N;in> \qquad (A.7)$$

where for the sake of simplicity we have omitted the spin indices S and S'.

Appendix B: $\gamma'\Delta' - \Delta$ vertex function with on mass shell Δ-s

Relations (3.19a,b) for the $\Delta - \gamma\Delta$ vertex functions are obtained from the spin 3/2 particle electromagnetic vertex function

$$< out; \mathbf{P}'|J^{\mu}(0)|\mathbf{P}; in >== (P+P')^{\mu}\Big(\overline{u}^{\sigma}(\mathbf{P}')\Big[g_{\rho\sigma}G_1(k'^2) + {k'}_{\sigma}{k'}_{\rho}G_3(k'^2)\Big]\Big)u^{\rho}(\mathbf{P})$$

$$+\overline{u}^{\sigma}(\mathbf{P}')\Big(-i\sigma^{\mu\nu}{k'}_{\nu}\Big[g_{\rho\sigma}G_2(k'^2) + {k'}_{\sigma}{k'}_{\rho}G_4(k'^2)\Big]\Big)u^{\rho}(\mathbf{P}), \qquad (B.1)$$

where ${k'}_{\mu} = (P-P')_{\mu}$ is the four-momentum of the emitted photon, $\mathbf{P} = \mathbf{p}_N + \mathbf{p}_{\pi} = \mathbf{P}_{\Delta}$, $P^o = \sqrt{m_{3/2}^2 + \mathbf{P}^2}$, $\mathbf{P}' = \mathbf{p}'_N + {\mathbf{p}'}_{\pi} = {\mathbf{P}'}_{\Delta}$, $P'^o = \sqrt{m_{3/2}^2 + \mathbf{P}'^2}$ are the four-momentum of a spin 3/2 particle with a mass $m_{3/2}$ in the initial and final states.

Extension in the complex mass and in the complex energy regions implies a transformations $m_{3/2} \Longrightarrow m_{\Delta} = M_{\Delta} - i/2\Gamma_{\Delta}$ and $k' \Longrightarrow k'_{\Delta}$. Here it is assumed that $M_{\Delta} = 1232MeV or 1210MeV$.

Functions $G_i(k'^2)$ in (B.1) are real. Therefore an important property of the electromagnetic Δ vertex functions (3.19a,b) is that the electromagnetic constants and $G_i(0)$ in (B.1) and in (3.19a,b) are also real.

The spin 3/2 particle vertex function (B.1) satisfy the one-particle Ward-Takahashi identity which in the complex energy region of Δ-resonance has the form

$$k'^{\mu}_{\Delta} < {\mathbf{P}'}_{\Delta}|J_{\mu}(0)|\mathbf{P}_{\Delta} >= \overline{u}^{\sigma}({\mathbf{P}'}_{\Delta})\Big[S^{-1}_{\sigma\rho}(P'^o_{\Delta}, {\mathbf{P}'}_{\Delta}) - S^{-1}_{\sigma\rho}(P^o_{\Delta}, \mathbf{P}_{\Delta})\Big]_{s_{\Delta}=s'_{\Delta}=m^2_{\Delta}} u^{\rho}(\mathbf{P}_{\Delta}), \qquad (B.2)$$

where $S_{\sigma\rho}(P'^o_{\Delta}, {\mathbf{P}'}_{\Delta})$ denotes a propagator of Δ [9, 10, 11].

It must be emphasized that extraction of the Δ degrees of freedom considered in this paper does not use a Heisenberg local field operator of the Δ resonance or a Lagrangian with the Δ degrees of freedom, because it is not possible to construct a Fock space for a "free" resonance state with a complex mass. In the present approach we operate only with vertex functions with on mass shell Δ's which are obtained in the limit $s \Longrightarrow m^2_{\Delta}$. Therefore ambiguities generated by unphysical gauge transformations of the Δ-particle field operator $\Psi^a_{\Delta} \longrightarrow \Psi^a_{\Delta} + C\gamma^a\gamma_b\Psi^b_{\Delta}$ [21] with an arbitrary parameter C does not appear in the present formulation. A sensitivity of the $\gamma p \to \gamma'\pi' p'$ observable on the choice of the form of the intermediate Δ propagator is demonstrated in [18].

In the off mass shell region one has four independent momenta of each Δ's because $P'^2 \neq m^2_{\Delta}$ and $P^2 \neq m^2_{\Delta}$. Therefore for the off mass shell Δ's (3.19a,b) takes more complicated form with increasing number of form factors G_i because each of the conditions $P^2_{\Delta} - m^2_{\Delta} \neq 0$ and $(i\gamma_{\sigma}P^{\sigma}_{\Delta} - m^2_{\Delta}) \neq 0$ doubling the number of form factors. Therefore instead of two form factors in (3.19b) we get 8 form factors for the off mass shell $\Delta - \gamma'\Delta'$ vertices. The role of these six additional formfactors is so important as important is the

off mass shell terms in the Δ propagator $S_{\sigma\rho}$. Besides these formfactors of the $\Delta-\gamma'\Delta'$ vertex with off mass shell Δ's are depending on three variables k'^2_Δ, P^2_Δ and P'^2_Δ. Therefore use of the off mass shell Δ propagators together with the on mass shell $\Delta-\gamma'\Delta'$ as it is done in refs. [9, 10, 11] is inconsistent.

Now we consider the recipe of extraction of the double Δ exchange term (3.18a) from the πN bremsstrahlung amplitude (2.6). According to our previous paper [17, 18], we consider s-channel part of the Green function τ_μ (2.2a) with the πN $''in''$ asymptotic states

$$\tau^\mu \Longrightarrow \sum_{\pi N,\pi' N'} <0|\mathsf{T}\Big(\Big[\Psi(y')\Phi(x')\Big]|\pi' N'; out><out;\pi' N'|\mathcal{J}^\mu(0)|\pi N; in><in;\pi N|\Big[\overline{\Psi}(y)\Phi^+(x)\Big]\Big)|0>$$

$$\Longrightarrow \sum_{\pi N,\pi' N'} <...\mathcal{G}_{\pi' N'}\Big[<out;\pi' N'|\mathcal{J}^\mu(0)|\pi N; in>\Big]_{\pi\ N\ irreducible}\mathcal{G}_{\pi N}...>, \qquad (B.3)$$

where $\mathcal{G}_{\pi N}$ denotes a full πN Green function. This Green function can be decomposed over the full system of the πN wave functions $|\psi_{\pi N}>$ which afterwards can be replaced by the Δ wave function $|\psi_\Delta>$

$$\mathcal{G}_{\pi N} = \sum_{\pi N} \frac{|\widetilde{\psi}_{\pi N}><\psi_{\pi N}|}{E-E_{\pi N}} \simeq \sum_{\Delta} \frac{|\widetilde{\psi}_{\Delta}><\psi_{\Delta}|}{E-E_{\Delta}}. \qquad (B.4)$$

Now if we take into account, that $<\Psi_{\Delta'}|\mathcal{J}^\mu(0)|\widetilde{\psi}_\Delta>$ is the vertex function (3.19a) and $<...|\Psi_{\pi' N'}>$ produces expression (3.16a), then we obtain (3.19a) after substitution of (B.4) into (B.3).

References

[1] F. E. Low, Phys. Rev. **110** (1958) 974.

[2] S. L. Adler and Y. Dothan, Phys. Rev. **151** (1966) 1267.

[3] W. E. Fischer and P. Minkowski, Nucl. Phys. **B36** (1972) 519.

[4] M. M. Musakhanov, Sov. J. Nucl. Phys. **19** (1974) 319.

[5] P. Pascual, and R. Tarrach, Nucl. Phys. **B134** (1978) 133.

[6] M. K. Liou and Z. M. Ding, Phys. Rev. **C35** (1987) 651.

[7] D. Lin and M. K. Liou, Phys. Rev. **C43** (1991) R930; Z. M. Ding, D. Lin and M. K. Liou, Phys. Rev. **C40** (1989) 1291.

[8] D. Lin, M. K. Liou and Z. M. Ding, Phys. Rev. **C44** (1991) 1819,

[9] M. El Amiri, G. Lopez Castro, and J. Pestieau. Nucl. Phys. **A543** (1992) 673

[10] Wen-Tai Chiang, V. Paskalutsa, Shin Nanan Yang and D. Drechsel. Phys. Rev.**C71** (2005) -15204

[11] V. Paskalutsa, M. Vanderhaeghen and Shin Nanan Yang. Phys. Rept. **427** (2007) 125.

[12] J. D. Bjorken and S.D.Drell, Relativistic Quantum Fields. (New York, McGraw-Hill) 1965.

[13] C. Itzykson and C. Zuber. Quantum Field Theory. (New York, McGraw-Hill) 1980.

[14] S. Brodsky, and R.W. Brown. Phys. Lett. **49** (1982) 966.

[15] S. Brodsky, and R.W. Brown. Phys. Rev. **D28** (1983) 624.

[16] A. I. Machavariani, Nucl. Phys. **403** (1983) 480.

[17] A. I. Machavariani and Amand Faessler. Ann. Phys. **309** (2004) 49.

[18] A. I. Machavariani and Amand Faessler. Phys. Rev.**C72** (2005) 024002.

[19] P. van Nieuwenhuizen, Phys. Rep. **68** (1981) 189.

[20] H.T.Williams, Phys. Rev. **C31** (1985) 2297.

[21] M. Bemmerrouche, R. M. Davidson and N. C. Mukhopadhyay, Phys. Rev. **C39** (1989) 2339 and references therein.

[22] S. Gasiorowicz. Elementary Particle Physics (John Wiley and Sons, New York-London-Sydney) 1966.

[23] R.E. Behrends and C. Fronsdal, Phys. Rev. **106** (1957) 345.

[24] M. I. Goldberger, M.Watson. Collision Theory (Wiley, New York-London-Sydney) 1965.

[25] M. A.B. Beg, B.W. Lee and A. Pais, Phys. Rev. Lett. **13** (1964) 514,

[26] H. Georgi. Lie Algebras in Particle Physics (Reading) 1982.

[27] M. A.B. Beg, and A. Pais, Phys. Rev. **137** (1965) B1514,

[28] B. M. K. Nefkens at al., Phys. Rev. **D18** (1978) 3911.

[29] L. Heller, S. Kumano, J. C. Martinez, and E. J. Moniz, Phys. Rev. **C35** (1987) 718.

[30] A. M. Bosshard at al., Phys. Rev. **D44** (1991) 1962; C. A. Meyer at al., Phys. Rev. **D38** (1988) 754.

[31] G. E. Brown, M. Rho, and V. Vento, Phys. Lett. **B97** (1980) 423.

[32] M. I. Krivoruchenko, Sov. J. Nucl. Phys. **45** (1987) 109.

[33] R. Wittman, Phys. Rev. **C37** (1988) 2075.

[34] G. Lopez Castro and I. A. Marino, Nucl. Phys. **697** (2002) 440.

[35] J. Franklin, Phys. Rev. **D66** (2002) 033010.